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Restriction without Quantification: Embedding and Probability for Indicative Conditionals

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Many modern theories of indicative conditionals treat them as restricted epistemic necessity modals. This view, however, faces two problems. First, indicative conditionals do not behave like necessity modals in embedded contexts, e.g., under 'might' and 'probably': in these contexts, conditionals do not contribute a universal quantification over epistemic possibilities. Second, when we assess the probability of a conditional, we do not assess how likely it is that the consequent is epistemically necessary given the antecedent. I propose a semantics which solves both problems, while still accounting for the data that motivated the necessity modal view. The account is based on the idea that the semantics of conditionals involves only a restriction of the relevant epistemic state, and no quantification over epistemic possibilities. The relevant quantification is contributed by an attitude parameter in the semantics, which is shifted by epistemic modals. If the conditional is asserted, the designated attitude is acceptance, which contributes a universal quantification, producing the effect of a restricted necessity modal.

Keywords: Indicative conditionals; Epistemic modals; Conditional probability; Adams' thesis; Expressivism; Attitudes; Dynamic semantics.

1. Introduction

Many modern theories of indicative conditionals treat them as restricted epistemic necessity modals. This family includes dynamic semantics accounts (Gillies 2004; 2009; Starr 2014a; Willer 2014; 2018) and semi-dynamic information-based accounts (Bledin 2014; Gillies 2009; 2010; Moss 2015; Punčochář & Gauker 2020;

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Yalcin 2007).[1] The central idea, which I will refer to as the Box View, is the following:[2]

Box View: the semantics of an indicative conditional $p \Rightarrow q$ involves two components:

1. Restriction: restrict the set of epistemic possibilities to the p-worlds;
2. Quantification: check that q is true at every world in the restricted set.

The conditional is true/accepted iff the check in the second step is successful.[3]

This view is motivated by a compelling analysis of how indicative conditionals are assessed, known as the *Ramsey test* view (after a remark in Ramsey 1929). The idea is that, in order to assess $p \Rightarrow q$ in an information state s, we proceed in two steps: first, we add the antecedent p to s, resulting in a hypothetical state $s[p]$; second, we check whether we accept the consequent q in this hypothetical state; if (and only if) so, the conditional is accepted in s.

In possible world semantics, an information state s determines a set of worlds, namely, those worlds which are possible according to the available information. Adding p to s corresponds to restricting to those worlds in which p is true; thus, the resulting hypothetical state is the set $s[p]$ of p-worlds in s. Checking whether p is accepted in this hypothetical state amounts to checking whether q is true in all the worlds in $s[p]$. Thus, the two steps of the Ramsey test procedure correspond exactly to the two components of the semantics of conditionals as postulated by the Box View.

The Box View of conditionals also makes a number of welcome predictions. For instance, it explains why a discourse like (1) sounds contradictory.

(1) If Alice left, she went to London; #but it might be that she left and went to Paris.

1. The restrictor account of Kratzer (1986) also fits broadly within this family, although due to its specific syntactic assumptions it needs to be discussed separately. We will do so in Section 6.

2. In this paper I focus on indicative conditionals, although the puzzles that I will raise concern subjunctive conditionals as well. The solution that I propose extends straightforwardly to the subjunctive case. However, making subjunctive assumptions involves a different hypothetical process than making indicative assumptions; since spelling out the details of this process is orthogonal to our main concerns, I will leave subjunctive conditionals mostly out of consideration here, coming back to them only in the conclusion section.

3. In some of the theories mentioned above, the view is implemented as giving truth conditions for conditionals relative to a world with an associated set of epistemic possibilities; in other theories, conditionals lack truth conditions, and the semantics delivers acceptance conditions relative to an information state.

However, the view also faces two serious problems. The first has to do with embeddings of conditionals under epistemic modals. When embedded, conditionals do not behave like necessity modals. For instance, consider (2):

(2) It might be that if Alice left she went to London.

Given the Box View, we would expect (2) to be a higher-order epistemic claim: it is possible that the consequent is necessary given the antecedent. This, however, is not what (2) means. What (2) means is simply that the consequent is possible given the antecedent—i.e., that if Alice left, she might have gone to London.

But how is this possible? If the conditional is a restricted necessity claim, how can the outer modal turn it into a restricted *possibility* claim? If the embedded conditional involves a universal quantification over antecedent worlds, how can the external modal prevent this quantification, and replace it with an existential quantification?[4]

The second problem, pointed out by Edgington (2014), concerns probability judgments.[5] Consider (3):

(3) If the coin was tossed, it landed heads.

If all we know is that the coin is fair, it is natural to judge (3) to be 50% probable. This is not the probability that it is epistemically necessary that the coin landed heads if tossed. That probability has got to be zero, since we know for sure that it is *not* epistemically necessary that the coin lands heads if tossed. This is unexpected: if a conditional is a restricted epistemic necessity claim, its probability should just be the probability that this epistemic necessity claim obtains.

In this paper, I propose an account of conditionals, modals, probabilities, and probabilistic vocabulary that achieves the following:

1. It accounts for the data that motivate the Box View. In particular, it predicts that a conditional $p \Rightarrow q$ is acceptable iff q follows on the supposition of p, vindicating the Ramsey test idea, and that $p \Rightarrow q$ is logically inconsistent with $\Diamond(p \wedge \neg q)$.

4. For recent discussion of this puzzle, see Gillies (2018).

5. See also Mandelkern (2018). A parallel problem for subjunctive conditionals is widely discussed in the literature (see in particular DeRose 1994; Edgington 2008; Moss 2013; Schulz 2014). In that case, the argument is directed at accounts that treat subjunctive conditionals as universal modal claims, though not ranging over epistemic possibilities. For an analogous point in the domain of future discourse, see Belnap, Perloff, and Xu (2001), Cariani and Santorio (2018).

2. It accounts for the peculiar way in which conditionals embed under epistemic modals. This is achieved without *ad-hoc* syntactic stipulations, in particular, without denying that conditionals can take scope with respect to modals.
3. It predicts that the probability of a factual conditional is the conditional probability of the consequent given the antecedent. Crucially, this result is achieved without *ad-hoc* stipulations about probabilities of conditionals. Rather, it is derived on the basis of the semantics of conditionals and of a general definition of probability applying to all sentences alike.

The account builds on the idea that epistemic sentences express attitudes towards a truth-conditional content, assessed relative to an information state s. The contribution of epistemic modals is that of indicating the relevant attitude. Thus, our treatment of epistemic modals fits within the expressivist line (see also Hawke & Steinert-Threlkeld 2021; Moss 2015; Willer 2013; Yalcin 2007; 2011), but it differs from its predecessors in that, as we shall see, the connection between epistemic modals and attitudes is reflected in the semantics in a more direct way.

If-clauses are devices for restricting the relevant information state: to express an attitude towards $p \Rightarrow q$ in state s is to express the same attitude towards q in the hypothetical state $s[p]$. Thus, conditionals are essentially devices to express conditional attitudes—attitudes subordinated to a supposition. This so-called *suppositional view* of conditionals has been defended in detail, notably by Edgington (1986; 1995; 2014), based on ideas from Adams (1975) (see also Bennett 2003). The view is widely appreciated for its psychological plausibility (as it links conditionals to the all-important process of *supposing*), its accurate empirical predictions (see, e.g., Evans & Over 2004), and its generality (as it extends in a natural way beyond statements, to an analysis of conditional questions and commands). However, acceptance of the suppositional view has been hindered by the lack of a precise compositional semantics. The present theory can be seen as filling this gap, supplying a specific formal implementation.

Crucially, on the proposed theory, the conditional operator does not contribute any quantification. Rather, what contributes the relevant quantification is the attitude expressed towards the conditional. I will propose that assertion is associated with the attitude of acceptance, which contributes a universal quantification over epistemic alternatives. In this way, it will turn out that the information conveyed by asserting a non-modal conditional $p \Rightarrow q$ is the one predicted by the Box View, i.e., that all p-possibilities are q-possibilities. However, this result is obtained by dividing labor: the restriction to p-worlds is contributed by the conditional operator, while the universal quantification is contributed by the acceptance attitude associated with assertion.

This difference is crucial when the conditional is not asserted, but occurs embedded under another operator, like 'might': in this case, the higher operator may shift the attitude being expressed away from acceptance. As a consequence, no universal quantification shows up in the semantics of the resulting sentence. As we will see, this provides a solution to the puzzle of embedded conditionals.

The situation is similar when the relevant conditional is not asserted, but assessed for probability. Since no acceptance attitude is involved in the process, no universal quantification over epistemic alternatives shows up. Instead, all the conditional contributes is the restriction to the antecedent worlds: assessing the probability of $p \Rightarrow q$ then amounts to assessing the probability of q relative to the p-worlds, i.e., to assessing the conditional probability of q given p. As we will see, this provides a solution to the puzzle of probabilities of conditionals.

Thus, the main conclusion of the paper is summarized in its title: although the Box View is right as a view about the acceptance conditions of indicative conditionals, it is wrong about how these conditions come about; what the conditional operator itself contributes is only a restriction of the relevant set of possibilities, and not also a quantification over the restricted set.

The paper is structured as follows. In Section 2 I discuss the considerations that motivate the Box View and the two problems it faces. In Section 3 I propose a new account of the semantics of indicative conditionals and epistemic modals. In Section 4 I show that the account solves the two problems under consideration, while retaining the attractions of the Box View. In Section 5 I discuss how to extend the system to capture objective readings of epistemic modals and conditionals, as well as the interaction of these constructions with negation. In Section 6 I discuss similarities and differences with Kratzer's restrictor theory. Section 7 concludes with some considerations on the proposal and prospects for future work.

2. The Box View: Attractions and Problems

In this section, I will spell out in more detail the attractive features of the Box View, which we will aim to preserve, and the problems it faces, which we will aim to overcome. I will use $\Rightarrow$ to denote the indicative conditional construction, and $\Diamond, \Box$, and P to denote the epistemic modals 'might', 'must', and 'probably'.[6]

6. That the conditional construction corresponds to a binary operator at logical form is a non-trivial assumption, which is challenged by Kratzer (1986). We will come back to this point in Section 6.

2.1. *Attractions*

There are several reasons for treating indicative conditionals as restricted epistemic necessity claims. One was discussed in the introduction: the Box View seems to arise in a natural way from the plausible Ramsey test view of how we assess conditionals.

A second reason is that a plain indicative conditional $p \Rightarrow q$ and a sentence like $p \Rightarrow \Box q$ containing an explicit epistemic 'must' in the consequent seem to convey exactly the same information (cf. Gillies 2010).

(4) a. If Alice left, she went to London. $p \Rightarrow q$
b. If Alice left, she must have gone to London. $p \Rightarrow \Box q$

The simplest explanation for this is that (4-a) and (4-b) mean exactly the same. If they do, since (4-b) is manifestly a restricted epistemic necessity claim, so is (4-a).

A third reason (cf. Yalcin 2018) is that a conditional $p \Rightarrow q$ sounds incompatible with the epistemic possibility claim $\Diamond(p \wedge \neg q)$ and anything stronger.

(5) If Alice left, she went to London; #but it might be that she left and went to Paris.

This is just what we would expect on the Box View: if $p \Rightarrow q$ conveys that all p-possibilities are q-possibilities, then it is incompatible with $\Diamond(p \wedge \neg q)$, which requires some p-possibility to be a $\neg q$-possibility. Conversely, if we want the incompatibility of $p \Rightarrow q$ and $\Diamond(p \wedge \neg q)$ to be accounted for semantically, the semantics of $p \Rightarrow q$ must be at least as strong as assumed by the Box View: for if it were possible for $p \Rightarrow q$ to be satisfied in some circumstance in which not all epistemically possible p-worlds are q-worlds, then that circumstance would satisfy both $p \Rightarrow q$ and $\Diamond(p \wedge \neg q)$, so the two would be consistent.

2.2. *Problem 1: Embedding*

When epistemic modals scope above conditionals, they seem to remove the universal force from the conditional and replace it with their own quantificational force. As an example, consider a conditional in the scope of an epistemic 'might':

(6) It might be that if Alice left she went to London. $\Diamond(p \Rightarrow q)$

Intuitively, (6) is a conditional possibility claim: it says that, among the epistemic possibilities where Alice left, there are some where she went to London.

This is not what the Box View would lead us to expect. According to that view, (6) should be a second-order epistemic statement: it is possible that, relative to the worlds where Alice left, it is necessary that she went to London.[7]

Moreover, (6) seems to convey exactly the same as (7) (cf. Gillies 2018).

(7) If Alice left, it might be that she went to London. $p \Rightarrow \Diamond q$

This, too, is puzzling from the perspective of the Box View: if conditionals contribute a universal quantification, then $\Diamond(p \Rightarrow q)$ should correspond to a $\exists\forall$ statement, while $p \Rightarrow \Diamond q$ should correspond to a $\forall\exists$ statement, so it is hard to see how the two could be equivalent.[8]

This phenomenon is not restricted to epistemic 'might'. For instance, consider conditionals embedded under a probability modal:

(8) It is probable that if Alice left she went to London. $\mathsf{P}(p \Rightarrow q)$

What (8) means is not that it is probable that Alice being in London is epistemically necessary on the supposition that she left; rather, what (8) means is just that Alice being in London is probable on the supposition that she left. Again, this does not involve any epistemic necessity; as in the case of 'might', the conditional does not seem to be contributing any universal quantifier over epistemic possibilities.

Again, a sentence like (8), where 'probably' embeds a conditional, sounds fully equivalent to (9), where 'probably' occurs in the consequent, which is unexpected if the conditional contributes a universal quantification.

(9) If Alice left, it is probable that she went to London. $p \Rightarrow \mathsf{P}q$

The point that we just illustrated for *might* and *probably* is quite general: analogous observations can be made about conditionals embedded under arbitrary

7. In many implementations of the Box View, the existential quantification associated with $\Diamond$ would go vacuous, so no proper second-order reading arises; but this does not improve the situation, since then $\Diamond(p \Rightarrow q)$ comes out as equivalent to $p \Rightarrow q$.

8. Could one respond by claiming that the relevant reading of (6) results from *might* taking narrow scope with respect to the *if*-clause at logical form? Two comments on that. First, if that is right, why don't we also have a second reading corresponding to surface scope? Of course, if both scopes lead to the same reading, we have an explanation. Second, we can look for ways to force wide scope for the possibility modal, and check if the relevant reading disappears. Consider someone saying:

(i) I'll go ahead and say something which I can't be sure of, but which I think is a possibility: if Smith stole the jewels, she left the country.

Here, the speaker appears to be explicitly claiming the epistemic possibility of a conditional. It seems really implausible that the possibility operator takes syntactic scope below the if-clause. Yet, the result is still unambiguously a claim of conditional possibility.

epistemic modals, including less studied ones. For instance, the following also sound equivalent:

(10) a. It is obvious that if Alice left she went to London.
b. If Alice left, it is obvious that she went to London.

Summing up, then, when embedded under epistemic modals, conditionals do not seem to contribute a universal quantification over epistemic alternatives. Moreover, conditionals seem to commute with epistemic modals. These facts are surprising from the perspective of the Box View analysis. This is not to say that one could not develop a theory that accounts for these observations while holding on to the Box View. But this does pose a challenge: existing Box View accounts do not provide a general explanation of these data, and it is not obvious how they could be modified to provide one.[9]

2.3. *Problem 2: Probability*

Imagine that a fair die has just been rolled, but the result has not been revealed yet. Our friend Alice makes the following claim:

(11) If the outcome is even, it is above three. $p \Rightarrow q$

Clearly, the claim is a guess—she cannot be sure of what she is saying. But it *is* quite likely. If we are asked to quantify just *how* likely, the natural answer

9. I am only aware of two attempts to account for the observations above while preserving some version of the Box View. The first is due to Gillies (2018), who is concerned specifically with the commutation of conditionals with *might*. Working in the framework of data semantics (Veltman 1981; 1985) Gillies proposes to fix the problem by devising a new entry for 'might' and claiming that epistemic 'might' is ambiguous between the standard and the revised entry. This proposal, however, is really specific to *might*; it is not clear how to extend it to a general story about the commutation of conditionals with other epistemic modals. The second account is due to Starr (2014b). In this account, the core semantics for conditionals is a dynamic implementation of the Box View; however, as a derivative notion, conditionals are also assigned trivalent truth-conditions, in line with Belnap (1970). Exploiting these trivalent truth-conditions, Starr manages to correctly predict that conditionals embedded under probability operators lead to claims of high conditional probability. Starr acknowledges, however, that his theory makes wrong predictions for conditionals embedded under *might*. In a footnote, he sketches a story that predicts the correct result if statements of the form 'it might be that if A then B' are interpreted not by the LF $\Diamond(p \Rightarrow q)$, but instead as involving a "truth" operator T intervening between the modal and the conditional, so that the LF we interpret is $\Diamond\,\mathsf{T}(p \Rightarrow q)$. In order for the explanation not to be an *ad-hoc* fix, more would have to be said about the source of the truth operator, and about why nestings of conditionals under *might* are obligatorily interpreted as involving this operator. Moreover, in this case, too, it is not clear how to extend the specific stories given for *might* and *probably* to a general story about arbitrary epistemic modals.

seems to be 2/3: for there are three equally likely even outcomes, and two of these are above three. In general, it seems that the way in which we attribute probabilities to conditionals conforms to the thesis of Adams (1975), according to which the probability of a conditional $p \Rightarrow q$ equals the conditional probability of q given p.[10]

This observation is puzzling from the perspective of the Box View. According to that view, what Alice has claimed is that in all epistemic possibilities in which the outcome is even, the outcome is above three. This is simply not the case: it is definitely possible that the die landed on two, and thus, that the outcome is even but not above three. Therefore, from the perspective of the Box View, it seems that Alice's claim should have no likelihood whatsoever.

To put it another way: according to the Box View, (11) means the same as (12).

(12) If the outcome is even, it must be above three. $p \Rightarrow \Box q$

But, unlike (11), (12) is *not* quite likely—it is not likely at all. But if (11) can be quite likely while (12) is not likely at all, surely (11) and (12) cannot mean the same thing![11]

It seems that, when we assess the probability of a conditional $p \Rightarrow q$, we do not estimate how likely it is that q is necessary given p; instead, we estimate how likely it is that q is *true* given p. From the perspective of the Box View, however, this estimate bears no obvious relation to the semantics of the conditional. This is undesirable: we would like to have an account of the semantics of conditionals that explains our probability intuitions about them. Since our intuitions seem to track the conditional probability of the consequent given the antecedent, we would like an account that predicts that, given what a conditional means, and

10. For an overview of the classical experimental literature, see Evans and Over (2004: §8). For an overview of the theoretical debate around the thesis, see Khoo and Santorio (2018).

11. To articulate this point a bit further: in theories based on the Box View, such as Yalcin (2007), Gillies (2010), Willer (2014; 2018), Starr (2014a; 2014b), two sentences can be logically equivalent—i.e., acceptable in the same states—without being semantically equivalent—without being assigned the same semantic value. This is the case, for instance, for the pair p and $\Box p$: although these two sentences are logically equivalent, they are not semantically equivalent, which opens the way to a definition of probability that treats them differently. However, in all the above theories the sentences $p \Rightarrow q$ and $p \Rightarrow \Box q$ are not just logically equivalent, but also semantically equivalent, since the universal quantification provided by $\Box$ is already incorporated in the entry for $\Rightarrow$. Thus, assuming the probability of a sentence is determined by its semantics—which seems a reasonable desideratum—it is impossible to pair these theories with a definition of probability that assigns different probabilities to $p \Rightarrow q$ and $p \Rightarrow \Box q$ —as it seems we should. By contrast, in the theory I propose below, the relation between $p \Rightarrow q$ and $p \Rightarrow \Box q$ is exactly the same as the relation between p and $\Box p$: the two are logically, but not semantically equivalent, and can therefore differ in probability value.

given a natural way to construe the notion of probability which is the subject of our intuitive judgments, the probability of $p \Rightarrow q$ just *is* the conditional probability of q given p.

3. Attitude Semantics

In this section I lay out a new account of the semantics of conditionals and epistemic modals. I will refer to this account as Attitude Semantics, abbreviated as AS.

Language. To present the theory explicitly, I will work with a formal language. The base layer of the language is a set $\mathcal{L}_0$ of *factual sentences*, the semantics of which can be given in terms of truth-conditions relative to possible worlds. For our purposes, it does not matter what $\mathcal{L}_0$ is, but for the sake of concreteness, I will take $\mathcal{L}_0$ to be the language of propositional logic based on a set $\mathcal{P} = \{p, q, \ldots\}$ of atomic sentences.

Definition 3.1 (Factual language, $\mathcal{L}_0$).

$\alpha ::= p \mid \neg\alpha \mid \alpha \wedge \alpha \mid \alpha \vee \alpha$ where $p \in \mathcal{P}$

The full language $\mathcal{L}$ that I will work with is obtained by enriching $\mathcal{L}_0$ with operators designed to capture epistemic vocabulary: a binary operator $\Rightarrow$ for the conditional construction; unary operators $\Box$ and $\Diamond$ for 'it must be that' and 'it might be that'; and a unary operator P for 'it is probable that'. To avoid further complications which are not essential to our concerns, we restrict to factual antecedents, and we do not consider Boolean compounds of epistemic sentences.[12]

Definition 3.2 (Epistemic language, $\mathcal{L}$).

$\varphi ::= \alpha \mid \alpha \Rightarrow \varphi \mid \Box\varphi \mid \Diamond\varphi \mid \mathsf{P}\varphi$ where $\alpha \in \mathcal{L}_0$

Models. I will assume as background a model $M = \langle W, V \rangle$ that provides a universe W of possible worlds together with a valuation function $V : \mathcal{P} \times W \to \{0,1\}$ which specifies the truth-values of atomic sentences at each possible world. In order to simplify the exposition I will assume that the set W is finite, although this restriction is not essential. The relation of *truth* between worlds $w \in W$ and

12. The reason to restrict to factual antecedents is that it is just not clear how the process of supposing epistemic sentences works (though see Kolodny & MacFarlane 2010 for a proposal that we may take on board). The reason not to look at compounds of epistemic sentences is that it is unclear, intuitively, how one should assign probabilities to sentences like $p \wedge (q \Rightarrow r)$ (Egré & Cozic 2011). In both cases, the complications are not caused by the particular assumptions of our semantics.

factual formulas α, denoted $w \vDash \alpha$, is defined as usual; I write $w(\alpha)$ for the truth value of α in w, and denote by $|\alpha|$ the set of worlds where α is true:

$$|\alpha| := \{w \in W \mid w \vDash \alpha\}$$

Information states. In order to spell out our semantics, we will need a formal notion of information states. Since we are concerned not just with qualitative notions, but also with probabilistic ones, I will take an information state to be a probability distribution on W. Since W is finite, we can represent such a distribution simply as a map which assigns a probability to each possible world.

Definition 3.3 (Information states).
An information state is a map $s : W \to [0,1]$ such that $\sum_{w \in W} s(w) = 1$.

A world w is *ruled out* by an information state s if it is assigned probability 0. Worlds which are not ruled out are referred to as the *live possibilities* in s.

Definition 3.4 (Live possibilities).
If s is an information states set of live possibilities is $L(s) := \{w \in W \mid s(w) \neq 0\}$.

The probability of a proposition $X \subseteq W$ is just the probability that X is true.

Definition 3.5 (Probability of a proposition). If $X \subseteq W$, $s(X) := \sum_{w \in X} s(w)$.

Suppositions. Next, we need a modeling of the process of making an indicative supposition. When we suppose α in a state s, we enter a new state $s[\alpha]$ in which α is treated as certain. Thus, all worlds in which α is false should be assigned probability 0. The relative probabilities of the α-worlds are unaffected by the supposition, and should just be rescaled by a factor $1/s(|\alpha|)$ so that they sum up to 1 again. In other words, supposing can be modeled by the operation of *conditionalization*. If α is ruled out in s, i.e., if s rules out all α-worlds, then the supposition cannot take place: we then say that α is not supposable in s.[13,14]

13. As an indicative assumption: one can of course suppose α as a subjunctive assumption, triggering a different kind of revision of the current state. That matters for subjunctive conditionals.

14. There might be reasons to allow for the possibility of supposing α as an indicative assumption even when α has probability 0 according to the state. This can be achieved by modeling an information state s, not as a standard probability function, but instead as a Popper function (Popper 1959), treating conditional probability as a primitive notion, rather than as defined by the ratio formula.

Definition 3.6 (Supposing). If s is an information state and α a factual sentence with $s(|\alpha|) \neq 0$, then $s[\alpha]$ is the information state defined as follows:

$$s[\alpha](w) = \begin{cases} \dfrac{s(w)}{s(|\alpha|)} & \text{if } w \in |\alpha| \\ 0 & \text{if } w \notin |\alpha| \end{cases}$$

Notice that the set of live possibilities after the supposition is $L(s[\alpha]) = L(s) \cap |\alpha|$.

Semantics. Traditionally, semantics is about specifying truth conditions for sentences in contexts. With much recent work (e.g. Gillies 2004; Hawke & Steinert-Threlkeld 2021; Moss 2015; Punčochář & Gauker 2020; Veltman 1996; Willer 2013; Yalcin 2007), we will assume that epistemic sentences, including sentences headed by epistemic modals as well as indicative conditionals, differ essentially from factual sentences, in that they do not have truth conditions relative to states of affairs. Rather, we will take such sentences to be devices for negotiating the epistemic attitude to be taken towards certain truth-conditional contents.[15] Accordingly, our semantics for the epistemic language $\mathcal{L}$ specifies what information it takes to bear a certain attitude to a sentence $\varphi \in \mathcal{L}$. More specifically, the semantics will take the form of a ternary relation between an information state s, an attitude A, and a sentence φ:

$$s \vDash_A \varphi$$

The possible values for the attitude parameter A include full acceptance (denoted $\forall$, and called simply 'acceptance' below), compatibility (denoted $\exists$), and partial acceptance to degree $x \in [0,1]$ (denoted π_x). Other values for the attitude parameter could be considered as well, but the ones above are sufficient for our present purposes.

Definition 3.7. The set of attitude parameters is $\mathsf{At} = \{\forall, \exists\} \cup \{\pi_x \mid x \in [0,1]\}$

We now have all ingredients to recursively specify the semantics.

For factual sentences $\alpha \in \mathcal{L}_0$, full acceptance in s amounts to s assigning credence 1 to the proposition expressed by α; compatibility amounts to non-zero credence in that proposition; and partial acceptance to degree x amounts to credence x or higher.

- $s \vDash_\forall \alpha \iff s(|\alpha|) = 1$
- $s \vDash_\exists \alpha \iff s(|\alpha|) \neq 0$

15. And for describing the properties of different bodies of information, as in the sentence "According to the detective, the butler might be the murderer." We will come back to this descriptive function of epistemic modals in Section 5.1.

- $s \models_{\pi_x} \alpha \iff s(|\alpha|) \geq x$

Notice that the first two clauses can be rewritten qualitatively as follows:

- $s \models_{\forall} \alpha \iff \forall w \in L(s) : w(\alpha) = 1$
- $s \models_{\exists} \alpha \iff \exists w \in L(s) : w(\alpha) = 1$

So, acceptance amounts to truth at all live possibilities, while compatibility amounts to truth at some live possibility.

The semantic role of an epistemic modal is that of indicating the attitude expressed towards the prejacent: acceptance for 'must', compatibility for 'might', and partial acceptance to a high degree for 'probably'. Formally, these operators work by shifting the attitude parameter: for every $A \in \mathsf{At}$,

- $s \models_A \Box\varphi \iff s \models_{\forall} \varphi$
- $s \models_A \Diamond\varphi \iff s \models_{\exists} \varphi$
- $s \models_A \mathsf{P}\varphi \iff s \models_{\pi_t} \varphi$ where $t \in [0,1]$ is a fixed threshold value

Finally, conditionals are interpreted by a generalized Ramsey test clause: to bear an attitude to the conditional is to bear the attitude to the consequent, on the supposition of the antecedent.

- $s \models_A \alpha \Rightarrow \varphi \iff s[\alpha] \models_A \varphi$

What if the antecedent α is not supposable in the state s? Then the conditional cannot be assessed at s. I take this to be a case of presupposition failure.[16] In this case we say that $\alpha \Rightarrow \varphi$ is *not admitted* by s.[17]

16. This is in line with a tradition that treats indicative conditionals as presupposing the epistemic possibility of their antecedent. See, among others, von Fintel (1998), Gillies (2009; 2010), Starr (2014a), Willer (2014; 2018).

17. The way to do this precisely is to define a notion of admittance of a sentence φ at a state s. Pre-theoretically, admittance corresponds to the fact that the presuppositions of φ are satisfied in s (see Heim 1988; Karttunen 1974). This is a pre-requisite for φ to be interpretable relative to s. In our setting, the only presupposition we take into account is the antecedent compatibility presupposition associated with indicative conditionals. Therefore, only sentences including conditionals can fail to be admitted at some contexts. The relation of admittance between a state s and a formula φ is defined recursively as follows:

- s admits α, if $\alpha \in \mathcal{L}_0$;
- s admits $\alpha \Rightarrow \varphi$ iff $s(|\alpha|) \neq 0$ and $s[\alpha]$ admits φ;
- s admits $\mathsf{M}\varphi$ iff s admits φ, for $\mathsf{M} \in \{\Box, \Diamond, \mathsf{P}\}$

Thus, for instance, $p \Rightarrow \Diamond(q \Rightarrow r)$ is admitted at s only in case $L(s)$ contains some $p \wedge q$-world.

Entailment and equivalence. Attitude semantics allows us to define different notions of entailment. A salient option is to define entailment as preservation of (full) acceptance (cf. Bledin 2014; Yalcin 2007): an entailment is valid if the conclusion is acceptable in every information state in which the premises are acceptable.[18,19]

Definition 3.8 (Logical entailment).
$\Phi \vDash \psi \iff \forall s : s \vDash_{\forall} \varphi$ for all $\varphi \in \Phi$ implies $s \vDash_{\forall} \psi$

The corresponding notion of equivalence tracks identity in acceptance conditions:

Definition 3.9 (Logical equivalence).
$\varphi \equiv \psi \iff \forall s : \left(s \vDash_{\forall} \varphi \iff s \vDash_{\forall} \psi \right)$

Notice that two formulas can be logically equivalent although they do not have the same semantics: this can happen if the acceptance conditions for the two formulas are the same, but their conditions relative to some other attitudes are different. To keep logical equivalence and semantic equivalence clearly distinct, it will be useful to introduce a notation, $\varphi \doteq \psi$, for the latter relation:

Definition 3.10 (Semantic equivalence).
$\varphi \doteq \psi \iff \forall s \forall A : \left(s \vDash_{A} \varphi \iff s \vDash_{A} \psi \right)$

Assertion. In the truth-conditional setting, an assertion of φ is normally construed, along the lines of Stalnaker (1978), as a proposal to the conversational participants to accept the proposition expressed by φ. In AS, not all sentences express propositions. However, the semantics gives us a notion

18. Taking into account admittance, this notion can be articulated in two versions. The strong version: whenever all the premises are admitted and accepted, the conclusion is also admitted and accepted. The weak version: whenever all the premises are admitted and accepted, if the conclusion is admitted, then it is also accepted. The latter is a notion of Strawson entailment in the sense of von Fintel (1999). These two notions are different: for instance, $p \Rightarrow p$ is valid in the weak sense (since whenever it is admitted, it is accepted) but not in the strong sense (since it is not always admitted). However, the entailment $\Diamond p \vDash p \Rightarrow p$ is valid even in the strong sense. For our purposes it does not matter which of the two versions we choose: all the entailment claims made below apply equally to both versions. The same holds for the notion of logical equivalence, defined in terms of entailment.

19. This is not the only interesting notion of consequence that we can define in AS. Following Adams (1965; 1975), we can also define a consequence relation $\vDash_{\pi}$ which preserves high probability:

- $\Phi \vDash_{\pi} \psi \iff \forall \epsilon > 0 \exists \delta > 0$ such that $\forall s : s \vDash_{\pi_{1-\delta}} \varphi$ for all $\varphi \in \Phi$ implies $s \vDash_{\pi_{1-\epsilon}} \psi$

of what it is to accept these sentences. Therefore, we do not need the detour through the proposition expressed. We can simply say that an assertion of φ is a proposal to the conversational participants to coordinate on a state which accepts φ.[20]

4. Predictions

Let us now turn to the predictions that AS makes. First, we will show that it can vindicate the conceptual considerations and the empirical observations that motivated the Box View of conditionals. Then, we will proceed to show how it solves the two problems discussed above.

4.1. Recovering the Predictions of the Box View

Consider the acceptance conditions that AS predicts for a conditional $p \Rightarrow q$ in a state where the antecedent is supposable. We have:

$$s \vDash_\forall p \Rightarrow q \iff s[p] \vDash_\forall q \iff L(s[p]) \subseteq |q| \iff L(s) \cap |p| \subseteq |q|$$

Thus, AS yields the same result as the Box View for the acceptance conditions of factual conditionals: $p \Rightarrow q$ is accepted in state s just in case all live p-possibilities in s are q-possibilities. Since an assertion is a proposal to adopt a state which accepts the sentence, the effect of asserting a factual conditional $p \Rightarrow q$ is also in accordance with the Box View. Notice moreover that AS fully vindicates the Ramsey test idea: accepting $p \Rightarrow q$ in a state s amounts to accepting q in the hypothetical state $s[p]$ resulting from the supposition of p.

We also predict the inconsistency of $p \Rightarrow q$ with $\Diamond(p \wedge \neg q)$. Indeed, we have:

$$s \vDash_\forall \Diamond(p \wedge \neg q) \iff s \vDash_\exists p \wedge \neg q \iff L(s) \cap |p \wedge \neg q| \neq \emptyset \iff L(s) \cap |p| \nsubseteq |q|$$

Thus, it is impossible for a state to accept simultaneously $p \Rightarrow q$ and $\Diamond(p \wedge \neg q)$. This can be stated as follows in terms of entailment.

20. To this basic effect of assertion we may add another: by asserting φ, the speaker undertakes a commitment to φ. Spelling this out would require giving a theory of the commitments induced by sentences of different forms. I will not spell out such a theory here, but it would be natural to take conditionals to induce conditional commitments: being committed to $\alpha \Rightarrow \varphi$ implies being committed to φ, if α turns out true. Thus, a speaker asserting $p \Rightarrow q$ may turn out to be factually right or wrong. See Edgington (1995: §7.3).

Fact 4.1. $p \Rightarrow q, \Diamond(p \wedge \neg q) \vDash \bot$

Finally, consider again:

(13) a. If Alice left, she went to London.
b. If Alice left, she must have gone to London.

AS makes these two sentences logically equivalent.

Fact 4.2. $p \Rightarrow q \equiv p \Rightarrow \Box q$

To see that the equivalence holds, notice that the two conditionals are admitted in the same states (those in which p is supposable), and when they are admitted, we have:

$$s \vDash_\forall p \Rightarrow q \iff s[p] \vDash_\forall q \iff s[p] \vDash_\forall \Box q \iff s \vDash_\forall p \Rightarrow \Box q$$

We can, thus, account for the fact that (13-a) and (13-b) can be inferred from each other, and that whenever one fully accepts one of these sentences, one also fully accepts the other. Moreover, given the connection between assertion and acceptance, we account for the intuition that (13-b) and (13-b) convey the same when asserted.

At the same time, $p \Rightarrow q$ and $p \Rightarrow \Box q$ are not semantically equivalent. To see this, consider the conditions under which these sentences are compatible with a state s:

- $s \vDash_\exists p \Rightarrow q \iff s[p] \vDash_\exists q \iff L(s[p]) \cap |q| \neq \varnothing$
- $s \vDash_\exists p \Rightarrow \Box q \iff s[p] \vDash_\exists \Box q \iff s[p] \vDash_\forall q \iff L(s[p]) \subseteq |q|$

Thus, consider a state s in which both $p \wedge q$ and $p \wedge \neg q$ are live possibilities. Such a state is compatible with $p \Rightarrow q$, but it is not compatible with $p \Rightarrow \Box q$.

4.2. Solving the Embedding Problem

We saw that both AS and the Box View predict $p \Rightarrow q$ to be acceptable just when all the epistemically possible p-worlds are q-worlds. But there is a crucial difference: in AS, the semantics of the conditional operator does not involve a universal quantification over epistemic possibilities. If we spell out the acceptance conditions of an unembedded conditional $p \Rightarrow q$, we can see that they do involve a universal quantification:

$$s \vDash_\forall p \Rightarrow q \iff s[p] \vDash_\forall q \iff \forall w \in L(s[p]) : w(q) = 1$$

But, as the derivation shows, the source of the universal quantifier is the acceptance attitude, not the conditional operator. When interpreting a conditional embedded under an epistemic modal, the relevant attitude may be shifted away from acceptance; as a result, no universal quantifier shows up in the semantics. For instance, consider the acceptance conditions for a conditional embedded under 'might'. Assuming p is supposable in s, we have:

$$\begin{aligned} s \vDash_{\forall} \Diamond(p \Rightarrow q) &\Leftrightarrow s \vDash_{\exists} p \Rightarrow q \\ &\Leftrightarrow s[p] \vDash_{\exists} q \\ &\Leftrightarrow \exists w \in L(s[p]) : w(q) = 1 \end{aligned}$$

Thus, we predict that $\Diamond(p \Rightarrow q)$ is not a second-order epistemic claim, but a simple claim of conditional possibility: q is possible conditionally on p. We can also predict that $\Diamond$ commutes with $\Rightarrow$: in fact, the formulas $\Diamond(p \Rightarrow q)$ and $p \Rightarrow \Diamond q$ are not just logically equivalent, but also semantically equivalent.

Fact 4.3. $\Diamond(p \Rightarrow q) \doteq p \Rightarrow \Diamond q$

Proof. Notice that the two sentences are admitted by the same states, namely, those in which p is supposable. For any such state s and any attitude A, we have:

$$\begin{aligned} s \vDash_{A} \Diamond(p \Rightarrow q) &\Leftrightarrow s \vDash_{\exists} p \Rightarrow q \\ &\Leftrightarrow s[p] \vDash_{\exists} q \\ &\Leftrightarrow s[p] \vDash_{A} \Diamond q \\ &\Leftrightarrow s \vDash_{A} p \Rightarrow \Diamond q \end{aligned}$$

Which shows that the two formulas are semantically equivalent. □

The story with conditionals embedded under 'probably' is completely analogous: P changes the attitude parameter to π_t, and thus the acceptance conditions for $\mathsf{P}(p \Rightarrow q)$ involve no universal quantification over epistemic possibilities. Instead, we get:

$$s \vDash_{\forall} \mathsf{P}(p \Rightarrow q) \;\Leftrightarrow\; s \vDash_{\pi_t} p \Rightarrow q \;\Leftrightarrow\; s[p] \vDash_{\pi_t} q \;\Leftrightarrow\; s[p](|q|) \geq t$$

Thus, $\mathsf{P}(p \Rightarrow q)$ is acceptable in case, in the state resulting from the supposition of p, the probability of q is high. As we will see, this means that the conditional probability of q given p is high. This is the intuitively correct prediction. Moreover, with a proof analogous to the one we saw for $\Diamond$, we can show that P and $\Rightarrow$ commute.

Fact 4.4. $\mathsf{P}(p \Rightarrow q) \doteq p \Rightarrow \mathsf{P}q$

Moreover, the account just given for $\Diamond$ and P extends straightforwardly to arbitrary epistemic modals, provided they are treated as shifters of the attitude parameter, by a semantic clause of the form $s \vDash_A \mathrm{M}\varphi \Leftrightarrow s \vDash a_{\mathrm{M}}\varphi$, where a_{M} is an attitude associated with M. Thus, AS provides a simple and general solution to the embedding problem: we can explain why conditionals embedded under epistemic modals do not contribute a universal quantification over epistemic possibilities, although the acceptance conditions for unembedded conditionals involve such a quantification; moreover, we have a general explanation for the commutation of epistemic modals with conditionals.

4.3. *Solving the Probability Problem*

How to characterize the probability $\mathbb{P}_s(\varphi)$ of a sentence relative to a state s? For factual sentences, this is clear: the probability of α is just the probability that α is true.

$$\mathbb{P}_s(\alpha) = s(|\alpha|)$$

Conditionals—we are assuming—don't express propositions, so they cannot be assigned probabilities in this way. Still, as we discussed, it seems that we can meaningfully assign probabilities to conditionals. Of course, we could follow Adams (1965; 1975) and simply stipulate that, for a factual conditional $\alpha \Rightarrow \beta$, its probability is just the conditional probability of the consequent given the antecedent. But this would not explain *why* we assign probabilities to conditionals in this way. Is there a sense in which, when we are estimating the probability of $p \Rightarrow q$, we are estimating *the same thing* as when we estimate the probability of q?

Moreover, this definition would not go far enough. Consider (14):

(14) If the die was rolled, then if the outcome was even, it was a six.

Given a fair die, it seems natural to say that (14) has probability $\frac{1}{3}$. But this conditional has the form $p \Rightarrow (q \Rightarrow r)$; since its consequent is not factual, it is not covered by the definition we are considering. Of course, we could add yet another *ad-hoc* clause for iterated conditionals, but it would be more satisfactory to find a single general notion that underlies all these particular cases.

Attitude semantics allows us to define such a general notion. We will take the probability of a sentence φ in a state s to be the highest degree to which φ is accepted in s (if φ is not accepted in s to any degree, we let the probability be zero). The idea is quite natural: the probability of a sentence is a measure of the degree to which a sentence is supported by the available information.

Definition 4.5 (Probability).
The probability of a sentence $\varphi \in \mathcal{L}$ in an information state s is the number:[21]

$$\mathbb{P}_s(\varphi) := \sup_{[0,1]}\left\{x \mid s \vDash_{\pi_x} \varphi\right\}$$

Let us look at the predictions that this makes. First, we can prove that, as we expect, the probability of a factual sentence α is the probability that α is true.

Fact 4.6 (Probabilities of factual sentences). If $\alpha \in \mathcal{L}_0$, $\mathbb{P}_s(\alpha) = s(|\alpha|)$
The proof is simple: $\mathbb{P}_s(\alpha) = \sup\left\{x \mid s \vDash_{\pi_x} \alpha\right\} = \sup\left\{x \mid s(|\alpha|) \geq x\right\} = s(|\alpha|)$.

Next, consider a factual conditional $p \Rightarrow q$. Based on the semantics of conditionals and the definition of probability, we can now *prove* Adams' thesis: provided the antecedent is supposable, the probability of $p \Rightarrow q$ is the conditional probability of q given p.

Fact 4.7 (Probabilities of conditionals). Let s be a state with $s(|p|) \neq 0$. Then:

$$\mathbb{P}_s(p \Rightarrow q) = s[p](|q|) = \frac{s(|p \wedge q|)}{s(|p|)}$$

Proof. $\mathbb{P}_s(p \Rightarrow q)$ is defined as the maximum x for which $s \vDash_{\pi_x} p \Rightarrow q$. Since

$$s \vDash_{\pi_x} p \Rightarrow q \iff s[p] \vDash_{\pi_x} q \iff s[p](|q|) \geq x$$

the maximum value of x for which this holds is $s[p](|q|)$. This gives the first identity. As for the second identity, we have:

$$s[p](|q|) \;=\; \sum_{w \in |q|} s[p](w) \;=\; \sum_{w \in |p| \cap |q|} \frac{s(w)}{s(|p|)} \;=\; \frac{1}{s(|p|)} \cdot \sum_{w \in |p \wedge q|} s(w) \;=\; \frac{s(|p \wedge q|)}{s(|p|)}$$

which completes the proof. □

Thus, now we can explain *why* the probability of a conditional is the conditional probability of the antecedent given the consequent in terms of our suppositional semantics of conditionals and our construal of probability. The reason is that all a conditional does is to restrict the evaluation state—and *not* introduce any quantification of its own: to accept $p \Rightarrow q$ to degree x in state s is just to accept

21. Given our semantics, if the set $\{x \mid s \vDash_{\pi_x} \varphi\}$ is non-empty the sup will actually be a maximum, i.e., it will be an element in the set. If the set $\{x \mid s \vDash_{\pi_x} \varphi\}$ is empty, the sup is 0. Thus, the only reason to formulate the definition in terms of 'sup' instead of 'max' is to avoid a definition by cases.

q to degree x in the restricted state $s[p]$. Therefore, the probability of $p \Rightarrow q$ in s is equal to the probability of q in $s[p]$, and this is just the conditional probability of q given p in s.[22]

Thus, AS provides a solution to the probability problem: we now have a definition of probability of a sentence which, given the semantics of conditionals, predicts that the probability of a conditional equals the conditional probability of the consequent on the antecedent.[23]

Notice that the same definition allows us to associate probabilities with iterated conditionals $\alpha \Rightarrow (\beta \Rightarrow \gamma)$. In AS, such a conditional is semantically equivalent with $\alpha \wedge \beta \Rightarrow \varphi$; in other worlds, the import-export equivalence is semantically valid.

Fact 4.8 (Import-export). $\alpha \Rightarrow (\beta \Rightarrow \varphi) \doteq \alpha \wedge \beta \Rightarrow \varphi$

Since the probability of a sentence in a state is defined in purely semantic terms, the probability of $\alpha \Rightarrow (\beta \Rightarrow \gamma)$ equals the probability of $\alpha \wedge \beta \Rightarrow \gamma$, which in turn is just the conditional probability of γ given $\alpha \wedge \beta$. This explains why it is natural to attribute probability $\frac{1}{3}$ to the conditional (14).

Finally, consider the modalized conditional $p \Rightarrow \Box q$. We have:

$$\begin{array}{lcl} s \vDash_{\pi_x} p \Rightarrow \Box q & \Leftrightarrow & s[p] \vDash_{\pi_x} \Box q \\ & \Leftrightarrow & s[p] \vDash_{\forall} q \\ & \Leftrightarrow & s[p] \vDash_{\forall} \Box q \\ & \Leftrightarrow & s \vDash_{\forall} p \Rightarrow \Box q \end{array}$$

22. On the idea that Adams' thesis can be explained in terms of the restricting role of if-clauses, see also Egré and Cozic (2011). The present work improves on that proposal in two ways. First, we give a general definition of probability of a sentence from which both the case of factual sentences and the case of conditionals follow as particular cases. Second, Egré and Cozic's approach deals with conditionals embedded under probabilistic modals (e.g., "There's a 50% chance that") but not with the probabilities of simple, unmodalized conditionals. Since Egré and Cozic build on Kratzer's restrictor view, they face the problem to be discussed in Section 6.

23. Kaufmann (2004) provides examples where we tend to judge a conditional as having a probability which is different from the conditional probability of the consequent given the antecedent. While these examples are very interesting, I do not think they witness failures of Adams' thesis, for reasons that I am not going to discuss here. However, suppose one is instead inclined to take these judgments as genuine violations of Adams' thesis. How could one accommodate these cases in the present theory? One could say that while conditionals always trigger a supposition of the antecedent, this supposition does not always happen by conditionalization; sometimes, a different procedure might be used. The semantic clauses and the definition of probability would stay the same. The solution to the embedding problem would still work; in particular, the commutation of conditionals and epistemic modals would still be predicted. Moreover, Adams' thesis would still be predicted to hold whenever suppositions happen by conditionalization.

Thus, for $p \Rightarrow \Box q$, probabilistic acceptance collapses to full acceptance: if $p \Rightarrow \Box q$ is fully accepted at s, then it is accepted to every degree $x \in [0, 1]$; and if it is not fully accepted, then it is not accepted to any degree $x \in [0,1]$.[24] This gives the following result:

$$\mathbb{P}_s(p \Rightarrow \Box q) = \begin{cases} 1 & \text{if } s[p] \subseteq |q| \\ 0 & \text{otherwise} \end{cases}$$

Thus, AS solves the remaining puzzle from Section 2.3: although the sentences $p \Rightarrow q$ and $p \Rightarrow \Box q$ have the same acceptance conditions, they are not in general associated with the same probability value. If some but not all of the p-possibilities in s are q-possibilities, then $p \Rightarrow q$ receives an intermediate probability value, while $p \Rightarrow \Box q$ receives probability 0. This explain the contrast in probability intuitions which we observed in Section 2.3 between (15-a) and (15-b).

(15) a. If the outcome is even, it is above three.
b. If the outcome is even, it must be above three.

Summing up, then, AS allows us to define a general notion of probability of a sentence which accounts for the probability intuitions discussed in Section 2.3. Moreover, the semantic characterization of this notion which yields the desired predictions is a conceptually natural one: probability is a measure of how strongly a sentence is supported by the available information.

4.4. Avoiding Triviality

There is a large literature on triviality results, which shows that Adams' Thesis along with some other assumptions about probabilities of conditionals leads to absurd conclusions (see Khoo & Santorio 2018 for a recent survey). The account of probabilities given by AS satisfies Adams' Thesis, yet it obviously does not lead to absurdity. Why? The reason is that the triviality results assume that the same formal properties that hold for probabilities of factual sentences also hold for probabilities of conditionals. This is not the case in AS: since probabilities of

24. This illustrates a general property of sentences involving epistemic modals: they are either fully accepted by the state, or incompatible with it (something similar holds for modal sentences in dynamic semantics: they are always either accepted or rejected by a state). I take this to be a good prediction: while it is possible to be in a state in which one is unsure about, say, whether it might be the case that p, this situation seems to require 'might' to be interpreted relative to a body of information other than the state of evaluation. Such cases can be modeled as involving the *anchoring operator* discussed in Section 5.1, which ties the modal to the relevant information source.

conditionals are not probabilities of propositions, they do not behave formally like probabilities of propositions. To give an example, many triviality results (starting with Lewis 1976) make crucial use of the following instance of the law of total probability, which requires:

$$\mathbb{P}_s(p \Rightarrow q) = \mathbb{P}_{s[r]}(p \Rightarrow q) \cdot \mathbb{P}_s(r) + \mathbb{P}_{s[\neg r]}(p \Rightarrow q) \cdot \mathbb{P}_s(\neg r)$$

provided $\mathbb{P}_s(r), \mathbb{P}_s(\neg r) > 0$. This identity is invalidated by the AS account of probabilities. In my view, this prediction is both empirically and conceptually motivated. Since there is much to say on this point, however, I will leave a detailed discussion for another occasion.[25]

Does this position amounts to a rejection of standard probability theory? In my view, it does not: instead, it calls for a more careful understanding of the role of probability theory. Probability theory is not concerned with language, but with the obtaining of certain "events", which on one interpretation can be identified with propositions. Thus, what probability theory gives us are constraints on the admissible ways to assign probabilities to propositions; what AS then gives us is an account of how to extend such an assignment of probabilities to sentences. On the present view, probabilities of sentences are not always probabilities of propositions, so we should not expect them to obey the same constraints as probabilities of propositions. Thus, the problem lies not with standard probability theory, but with the way it is sometimes construed. Probability theory is a theory of the probabilities of *propositions*, not *sentences*.

To conclude, let me discuss a common objection to views like the present one. According to this objection, the axioms of probability theory are constitutive of the concept of probability; therefore, if a certain quantity that attaches to sentences does not obey these axioms, that quantity cannot properly be called *probability*.[26]

25. A reviewer asks, quite naturally, whether the map $\mathbb{P}_S(\cdot)$ satisfies (an algebraic counterpart of) the Kolmogorov axioms, and whether the conditional probability of φ given α, captured by $\mathbb{P}_{s[\alpha]}(\varphi)$, always coincides with the result of the ratio formula $\mathbb{P}_S(\alpha \wedge \varphi) / \mathbb{P}_S(\alpha)$. Since our semantics does not interpret Boolean compounds of conditionals, these questions cannot be properly formulated. However, we can see from the failure of the above principle that any extension of the present account that does interpret such compounds is bound to invalidate either the Kolmogorov axioms or the ratio formula for conditional probability when these are applied to conditionals. For if one had both, the above principle would hold. See Bradley (2006) for an argument to the effect that, indeed, one of these principles should be invalidated to get plausible predictions about probabilities of conditionals.

26. The objection goes back to Lewis, who made this point against Adams (1975):

> But if it be grated that the 'probabilities' of conditionals do not obey the standard laws, I do not see what is to be gained by insisting on calling them 'probabilities'. (Lewis 1976: 304–305)

I have two things to say in response. First: what we call the magnitude $\mathbb{P}_s(\varphi)$ does not matter to the success of the theory. Those who insist that the term *probability* be treated as a term of art are welcome to call $\mathbb{P}_s(\varphi)$ the *likelihood* of φ.[27] What does matter is whether this quantity is what speakers estimate when they judge certain sentences to be "probable", "highly unlikely", "60% probable", and so on. For ultimately, what we want is for the theory to give a satisfactory account of our linguistic practices.

Second, I think it is in fact justified to call $\mathbb{P}_s(\varphi)$. the probability of φ in s. In my view, *probability*, like other key notions like *meaning* or *entailment*, is not first and foremost a term of art, but an informal concept susceptible of different formal reconstructions, which lead to more or less successful accounts of how language works. What I have proposed here is such a reconstruction. In a similar vein, it is natural to view, say, dynamic semantics as proposing a formal reconstruction of the notions of meaning and consequence, which is different from that given by truth-conditional semantics, and has different formal features, but which still aims to account for how we use language in communcation and inference.

5. Extensions

5.1. Anchoring

We treated epistemic modals as devices to express attitudes towards truth-conditional contents. Sometimes, however, epistemic modals, like other modals, are used to describe facts about the world. For instance, (16) seems to be a descriptive statement about the information available to Alice.

(16) According to Alice, the butler might be the murderer.

Some theories of epistemic vocabulary (e.g., Moss 2015) account for this by postulating that epistemic modals are ambiguous between a factual reading and an expressive reading. Attitude semantics provides an elegant way of capturing these truth-conditional occurrences of epistemic modals without postulating such an ambiguity. To achieve this result, we will introduce a new operator, the *anchoring* operator, which captures the semantic effect of locutions like "according to Alice". This operator is in fact needed independently of epistemic modals, to account for sentences such as the following.[28]

27. Thanks to Matt Mandelkern for this terminological suggestion.

28. Thanks to Shane Steinert-Threlkeld for drawing my attention to this independent motivation.

(17) According to Alice, the butler is the murderer.

We will enrich our formal language with a set $\mathcal{A}$ of labels, standing for different sources of information: Alice's information, the content of a certain database, the results of a medical examination, etc. We will refer to these labels as *anchors*. We also extend our models with a family of maps $\{s_a \mid a \in \mathcal{A}\}$ from worlds to information states, where $s_a(w)$ represents the information state associated to source a at world w.

Given a sentence $\varphi \in \mathcal{L}$ and an anchor $a \in \mathcal{A}$, we will be able to form a new factual sentence $\downarrow_a \varphi \in \mathcal{L}_0$, standing for the claim "according to a, φ". Since sentences of the form $\downarrow_a \varphi$ are factual, their semantics will be given in terms of truth-conditions with respect to worlds. The relevant truth conditions are simple: "according to a, φ" is true at w iff the information state associated with a at w accepts φ.

Definition 5.1 (Semantics of the anchoring operator).

- $w \vDash \downarrow_a \varphi \iff s_a(w) \vDash_\forall \varphi$

Let us illustrate the clause by looking at the examples above, repeated below:

(18) a. According to Alice, the butler is the murderer. $\downarrow_a p$
b. According to Alice, the butler might be the murderer. $\downarrow_a \Diamond p$

Our semantics delivers the following truth-conditions for these sentences:

- $w \vDash \downarrow_a p \iff s_a(w) \vDash_\forall p \iff L(s_a(w)) \subseteq |p|$
- $w \vDash \downarrow_a \Diamond p \iff s_a(w) \vDash_\forall \Diamond p \iff s_a(w) \vDash_\exists p \iff L(s_a(w)) \cap |p| \neq \varnothing$

So, (18-a) is true if Alice's information implies that the butler is the murder, while (18-b) is true if Alice's information is compatible with the butler being the murder. These are good predictions.

Notice that, in the interpretation of (18-b), our clause for epistemic modals as attitude shifters is exploited to produce a truth-conditional result. Thus, the idea that epistemic modals are shifters of the attitude parameter does not stand in contrast to the fact that epistemic modals have objective, truth-conditional readings; instead, it can be exploited to derive these readings.

Also, crucially, (18-b) is correctly predicted to involve not two epistemic quantifiers—one associated with the anchoring and one with the modal—but a single quantifier, whose kind (existential) is determined by the modal, and whose range (Alice's state) is determined by the anchor. This elegant interplay is made possible by the fact that no universal quantifier is directly built into the meaning of the anchoring operator. Rather, the anchoring operator by default sets the attitude parameter to acceptance; if no modal operator interferes, as in (18-a), the acceptance parameter ultimately provides a universal quantifier over epistemic alternatives; however, if a modal operator is present, as in (18-b), it may shift the attitude parameter away from acceptance; as a consequence, no universal quantification will show up in the resulting reading. Thus, the same fundamental idea which allowed us to solve the problem of embedded conditionals also does some work in other places, such as the interpretation of modals in anchored contexts.

Notice that all the phenomena concerning conditionals that we discussed above also arise when conditionals occur in an anchored context. For instance, (19-a) and (19-b) seem to mean exactly the same.

(19) a. According to Alice, it might be that if Bob left he went to London.
$\downarrow_a \Diamond(p \Rightarrow q)$

b. According to Alice, if Bob left it might be that he went to London.
$\downarrow_a (p \Rightarrow \Diamond q)$

The explanations that we saw for the un-anchored case all carry over. For instance, (19-a) and (19-b) are indeed predicted to be semantically equivalent. Here is the proof that they have the same truth-conditions, where the second equivalence uses Fact 4.3:

$$\begin{array}{lcl} w \vDash \downarrow_a \Diamond(p \Rightarrow q) & \Leftrightarrow & s_a(w) \vDash_\forall \Diamond p \Rightarrow q \\ & \Leftrightarrow & s_a(w) \vDash_\forall p \Rightarrow \Diamond q \\ & \Leftrightarrow & w \vDash \downarrow_a (p \Rightarrow \Diamond q) \end{array}$$

This shows that the solution to the embedding problem that AS provides does not crucially rest on treating modal sentences as lacking truth conditions; rather, it rests on something more abstract, namely, the presence of a specific state-based layer in the semantics, where modals operate as shifters of an attitude parameter, and conditionals as restrictors of an information state parameter. In principle, the proposal is even compatible with the view that the state-based layer is "subordinate" to the truth-conditional layer and only ever triggered by (possibly covert) anchoring operators.[29]

29. Notice however that this kind of view, unlike the one advocated here, still faces the probability problem.

Occurrences of modals can be anchored not just when the sentence contains an explicit 'according to' locution, but also when an anchor is implicit. This allows us to make sense of the possibility of nestings of epistemic modals. E.g., suppose we observe a detective at work, not knowing exactly what evidence she has gathered. It seems that, in this context, we can use (20) to convey that the detective might have gathered enough evidence to conclude that the butler did it.

(20) It might be that the butler must be the murderer.

This can be predicted if we analyze (20) as involving an implicit anchor to the detective's information for the second modal: indeed, the formula $\Diamond \downarrow_a \Box p$ is accepted in s just in case s is compatible with the truth of $\downarrow_a \Box p$, i.e., with the possibility that according to the detective the butler must be the murderer.

Notice that, intuitively, a reading where at least one modal is anchored seems to be the only option for (20). Here is a possible explanation: in AS, iterating epistemic modals is always redundant in the absence of an anchor, since later modals overwrite the parameter setting which the earlier modals contribute. Assuming, with various authors (e.g., Katzir & Singh 2013; Meyer 2013) that natural languages implement a pragmatic ban against structural redundancy, this renders the LF without anchoring pragmatically unavailable for (20).

Finally, let me briefly note that the strategy described in this section can also be used to give an account of attitude verbs. The idea would be that an attitude verb operates similarly to the anchoring operator above: it selects its own information state, its own quantificational force, and checks whether the prejacent is supported at that state with that force. It seems natural to think, for instance, that attitude verbs like *believe*, *suspect*, and *doubt* operate by selecting the same state—the agent's doxastic state—but different attitudes. Of course, a proper development of the idea must be left for another occasion.[30]

30. A reviewer also suggests the intriguing hypothesis that an attitude verb like *want* uses the same information state as *believe*, but a different attitude parameter, whose semantics involves, say, preferential relations between worlds in the state. They point out that this provides a principled explanation for Heim's observation that a desire ascription requires the presupposition of its prejacent to be satisfied by the agent's belief state (Heim 1992). They further suggest a starting point for an explanation for why epistemic modals do not embed under *want* (Anand & Hacquard 2014): for if such a modal were present, the specific contribution of *want* with respect to *believe* would be obliterated. This are very interesting ideas, which should explored in more detail in future work.

5.2. *Negation*

Let us now turn to negation. Though not a modal operator, various observations suggest that negation also commutes with the conditional, i.e., that $\neg(p \Rightarrow q)$ is equivalent to $p \Rightarrow \neg q$. This is hard to check directly, since conditionals may only be negated by means of circumlocutions like "it is not the case that", which may also express a more pragmatic kind of rejection. However, the situation becomes clearer if we embed conditionals under linguistic items which arguably lexicalize negation, such as the attitude verb 'doubt' or the quantifier 'nobody'. Following several authors (von Fintel & Iatridou 2002; Higginbotham 1986; Klinedinst 2010; Santorio 2017), consider:

(21) a. I doubt that if Alice took the test she passed. $B\neg(Ta \Rightarrow Pa)$
b. I believe that if Alice took the test she failed. $B(Ta \Rightarrow \neg Pa)$

(22) a. Nobody passed if they took the test. $\forall x\neg(Tx \Rightarrow Px)$
b. Everybody failed if they took the test. $\forall x(Tx \Rightarrow \neg Px)$

The two sentences of each pair seem to convey the same. Assuming that 'doubt' is equivalent to 'believe not', 'nobody' to 'everyone not', and 'fail' to 'not pass', these equivalences can be accounted for, provided that $\neg(p \Rightarrow q) \equiv p \Rightarrow \neg q$.

An asset of attitude semantics is that it can be extended in a natural way with an account of negation that predicts this commutation. Let us see how.

First, we extend the syntax given in Definition 3.2 by allowing all formulas of our epistemic language $\mathcal{L}$ to be negated. To interpret negation beyond the factual fragment, we need to extend our semantic machinery: following so-called *bi-lateral theories*,[31] we assume that, relative to a state s and an attitude A, a formula φ can be supported either positively ($s \vDash_A^+ \varphi$) or negatively ($s \vDash_A^- \varphi$). The semantics given above can be extended in an elegant way to this bi-lateral setting.

Let us start from factual formulas $\alpha \in \mathcal{L}_0$. In this case, the attitude parameter specifies the relevant kind of quantification over the live possibilities, while the polarity parameter $(+/-)$ specifies whether we are concerned with truth or with falsity.

31. Examples include data semantics (Veltman 1981; 1985) and versions of dynamic semantics (Willer 2018), inquisitive semantics (Bledin 2020), and truth-maker semantics (Fine 2017).

- $s \vDash^{+}_{\forall} \alpha \Leftrightarrow \forall w \in L(s): w(\alpha) = 1$
- $s \vDash^{-}_{\forall} \alpha \Leftrightarrow \forall w \in L(s): w(\alpha) = 0$
- $s \vDash^{+}_{\exists} \alpha \Leftrightarrow \exists w \in L(s): w(\alpha) = 1$
- $s \vDash^{-}_{\exists} \alpha \Leftrightarrow \exists w \in L(s): w(\alpha) = 0$
- $s \vDash^{+}_{\pi_x} \alpha \Leftrightarrow s(\{w | w(\alpha) = 1\}) \geq x$
- $s \vDash^{-}_{\pi_x} \alpha \Leftrightarrow s(\{w \mid w(\alpha) = 0\}) \geq x$

Next, take modals: as before, modals act as shifters of the attitude parameter; moreover, they behave in a dual way on the positive and negative side.

- $s \vDash^{+}_{A} \Box\varphi \Leftrightarrow s \vDash^{+}_{\forall} \varphi$
- $s \vDash^{-}_{A} \Box\varphi \Leftrightarrow s \vDash^{-}_{\exists} \varphi$
- $s \vDash^{+}_{A} \Diamond\varphi \Leftrightarrow s \vDash^{+}_{\exists} \varphi$
- $s \vDash^{-}_{A} \Diamond\varphi \Leftrightarrow s \vDash^{-}_{\forall} \varphi$
- $s \vDash^{+}_{A} \mathsf{P}\varphi \Leftrightarrow s \vDash^{+}_{\pi_t} \varphi$
- $s \vDash^{-}_{A} \mathsf{P}\varphi \Leftrightarrow s \vDash^{-}_{\pi_{1-t}} \varphi$

As before, conditionals simply restrict the evaluation state to the antecedent worlds, leaving everything else the same.[32]

- $s \vDash^{+}_{A} \alpha \Rightarrow \varphi \Leftrightarrow s[\alpha] \vDash^{+}_{A} \varphi$
- $s \vDash^{-}_{A} \alpha \Rightarrow \varphi \Leftrightarrow s[\alpha] \vDash^{-}_{A} \varphi$

Finally, negation flips positive and negative support:

- $s \vDash^{+}_{A} \neg\varphi \Leftrightarrow s \vDash^{-}_{A} \varphi$
- $s \vDash^{-}_{A} \neg\varphi \Leftrightarrow s \vDash^{+}_{A} \varphi$

Now let us look briefly at how negation behaves in this system.

First, since we also have negation as a truth-conditional operator, in principle we might have a conflict between the result we get when we interpret negation at the truth-conditional level, as reversing the truth-value of a formula at each world, and the result we get when we interpret it at the support level, as reversing the polarity of support. However, one can check that, in fact, the two interpretations lead to the same results. For instance, consider the case of positive acceptance. If we interpret $\neg$ as a truth-conditional operator, we have:

32. As before, we take $s[\alpha]$ to be defined only α is supposable, and we take conditionals to be interpretable in a state only if this condition is satisfied.

$$s \models^{+}_{\forall} \neg\alpha \iff \forall w \in L(s): w(\neg\alpha) = 1 \iff \forall w \in L(s): w(\alpha) = 0$$

If we interpret $\neg$ at the support level, we get to the same result:

$$s \models^{+}_{\forall} \neg\alpha \iff s \models^{-}_{\forall} \alpha \iff \forall w \in L(s): w(\alpha) = 0$$

Second, negation interacts elegantly with the modalities: indeed, $\Box$ and $\Diamond$ are semantically dual to each other via negation in the usual way.

Fact 5.2 (Duality) For every formula $\varphi \in \mathcal{L}$:

- $\Box\varphi \doteq \neg\Diamond\neg\varphi$
- $\neg\Box\varphi \doteq \Diamond\neg\varphi$
- $\Diamond\varphi \doteq \neg\Box\neg\varphi$
- $\neg\Diamond\varphi \doteq \Box\neg\varphi$

As an illustration, here is the proof that the formulas $\Box\varphi$ and $\neg\Diamond\neg\varphi$ have the same positive support conditions with respect to every attitude (the proof that they also have the same negative support conditions is completely analogous):

$$\begin{array}{lcl} s \models^{+}_{A} \neg\Diamond\neg\varphi & \iff & s \models^{-}_{A} \Diamond\neg\varphi \\ & \iff & s \models^{-}_{\forall} \neg\varphi \\ & \iff & s \models^{+}_{\forall} \varphi \\ & \iff & s \models^{+}_{A} \Box\varphi \end{array}$$

Finally, negation commutes with the conditional.

Fact 5.3. $\neg(p \Rightarrow q) \doteq p \Rightarrow \neg q$

Here is the proof. Again, I spell out only the case of positive support, but the case of negative support is completely analogous.

$$\begin{array}{lcl} s \models^{+}_{A} \neg(p \Rightarrow q) & \iff & s \models^{-}_{A} p \Rightarrow q \\ & \iff & s[p] \models^{-}_{\forall} q \\ & \iff & s[p] \models^{+}_{\forall} \neg q \\ & \iff & s \models^{+}_{A} p \Rightarrow \neg q \end{array}$$

6. Comparison with the Restrictor Theory

According to Kratzer's restrictor theory of conditionals (Kratzer 1986) the embedding problems that we faced in Section 2.2 stem from a fundamental mistake about the syntax of conditionals: there is no operator $\Rightarrow$ corresponding to the 'if ... then' construction in natural language. Rather, *if*-clauses spell out the restrictor argument of a modal operator. Thus, for instance, the following sentences do not have the logical forms we have assumed above, but rather the ones given on the right:

(23) a. If Alice left, she must have gone to London. $\Box_p q$
b. If Alice left, she might have gone to London. $\Diamond_p q$
c. If Alice left, she probably went to London. $\mathsf{P}_p q$

In these formulae, the modified operator O_p works like the original operator O, except that the relevant domain is restricted to worlds where p is true.

Moreover, the sentences in (24), though differering in the way the operators appear at surface form, actually correspond to exactly the same logical forms.

(24) a. It must be that if Alice left she went to London. $\Box_p q$
b. It might be that if Alice left she went to London. $\Diamond_p q$
c. It is probable that if Alice left she went to London. $\mathsf{P}_p q$

Thus, on this approach the problem of why the sentences in (23) sound equivalent to the sentences in (24) vanishes—or, rather, it is transformed from a semantic problem to a problem of syntax-semantics interface.

What about plain conditionals like (25), so-called *bare* conditionals?

(25) If Alice left, she went to London. $\Box_p q$

Kratzer postulates that such a sentence actually contains a silent epistemic 'must', so its logical form is the same as that of (23-a) and (24-a), namely, $\Box_p q$. Thus, the equivalence between 'if A, C' and 'if A, must C' is obtained by stipulation.

Finally, Kratzer's theory can explain why, although (25) involves a universal quantification over epistemic possibilities, (24-b) does not. According to this theory, appearances are deceiving: (24-b) is not obtained by embedding (25) under 'might', but by replacing the silent 'must' in the logical form of (25) by a 'might'. As a result, the logical form of (25) does not occur as a sub-constituent in (24-b). In this way, the restrictor theory dissolves the embedding problem discussed in Section 2.2.

However, the theory still faces the probability problem. To see why, consider again the scenario described in Section 2.3. A fair die was rolled and the outcome has not yet been revealed. Our friend Alice makes the following guess:

(26) If the outcome is even, it is above three. $\Box_p q$

Intuitively, though she cannot be sure, what she said is quite likely, since two out of three even outcomes are above three. But now, if the logical form of (26) is $\Box_p q$, what Alice has asserted is that it is epistemically necessary that the outcome is above three, given that it is even. Since this is clearly not the case, we should judge what she said to have probability zero or near-zero. But that seems wrong.

It is sometimes suggested in discussions that the advocate of the restrictor theory could respond by giving the following error theory: when we say that the probability of (26) is 2/3, what we are in fact doing is not judging the probability of the bare conditional (26); instead, we are making an assertion of the form "It is 2/3 probable that if the outcome is even, it is above three", in which the *if*-clause restricts the explicit probability operator. But this does not solve the problem, since it does not explain why we would assert this when asked to judge the likelihood of what Alice said. According to the theory, this behavior is perplexing: what Alice said is $\Box_p q$, but when we are asked how likely that is, we instead make an assertion about the conditional probability of q given p. Why would we do that? The given error theory does not seem to offer a coherent explanation of the way we behave when asked about the probabilities of conditionals (for related discussion of this problem, see also Mandelkern 2018).[33]

Let us now examine some similarities and differences between Kratzer's restrictor theory and attitude semantics. An important similarity is that, in both theories, conditional constructions serve only a restricting function, and do not contribute a quantifier. The source of quantification lies elsewhere. This fundamental point of convergence is what allows both theories to predict, e.g., that the sentence "it might be that if A then C" involves just an existential quantifier over possible worlds, and no universal quantifier.

At the same time, there is a key difference between the two theories. The source of quantification is not the same in both: in Kratzer's theory, the source of quantification is a modal operator; therefore, we need to assume that a modal operator is always present in a sentence involving conditionals, whence the

33. These considerations might raise the question of what, according to our own account, is the object whose probability we are judging—given that it is not a proposition. The quick answer is that it is the content expressed by Alice's utterance. This content can be identified with a set of state-attitude pairs, namely, $[\![p \Rightarrow q]\!] = \{\langle s, A\rangle \mid s \models_A p \Rightarrow q\}$. This means that the present approach yields a generalized notion of sentential contents, suitable to capture both contents of factual and non-factual sentences. See Ciardelli (in press) for detailed discussion of this point.

need to postulate silent necessity modals in the logical form of bare conditionals. Besides lacking independent motivation, this stipulation is empirically problematic, as we saw, in light of probability judgments about conditionals. In AS, by contrast, the source of quantification is the attitude parameter; while modal operators can shift this parameter, we need not assume that every sentence contains a modal operator. Thus, AS obviates the need for covert modals. If a sentence does *not* contain an epistemic modal that fixes the attitude parameter, the relevant quantification is not determined once and for all from within the sentence, but is determined from the outside, by the attitude under consideration. If the sentence is asserted, the relevant attitude is full acceptance, which is responsible for introducing a universal quantifier, producing the same effect as if the sentence had contained a 'must'. But in case the sentence is assessed for probabilistic acceptance, the process will involve weighing of probabilities rather than universal quantification, thus differing from the assessment of the corresponding 'must' sentence. This is why attitude semantics, unlike Kratzer's version of the restrictor theory, can solve the probability problem.

To conclude, let me point out that, while differing from Kratzer's specific theory, attitude semantics can itself be seen as embodying a version of the restrictor view of conditionals: *if*-clauses are merely devices to restrict the relevant set of epistemic possibilities. This may seem too restrictive: as Kratzer (1981) emphasized, in general *if*-clauses can serve to restrict different things, not just a set of epistemic possibilities. But while the simple theory presented in this paper is not equipped with the resources to deal with *if*-clauses which perform non-epistemic restrictions, it can be extended with such resources in a natural way, drawing on ideas from the restrictor tradition. Such an extension is described in Ciardelli (in press).

7. Conclusion

According to the Box View which lies at the core of many modern accounts, indicative conditionals are essentially restricted epistemic necessity modals. Two kinds of considerations challenge this view: first, conditionals do not embed under other operators like epistemic necessity modals; and second, the probability of a conditional is not the probability of an epistemic necessity claim. Both observations point to the conclusion that the core semantics of conditionals involves only a restriction of the set of relevant alternatives, and not a universal quantification over the resulting set.

This simple idea, however, is not easy to turn into a concrete proposal. After all, suppose we have used the antecedent to restrict the relevant set, generating a hypothetical information state; what should we then do with the consequent?

The standard answer is: check whether it is acceptable—whence the universal quantification. In this paper we saw that another answer is possible: do with the consequent whatever you were originally doing with the conditional. If you were checking if the conditional is acceptable, check if the consequent is acceptable in the hypothetical state; if you were judging the probability of the conditional, judge the probability of the consequent in the hypothetical state; and so on.

We have seen how to implement this idea in a formal system which we called *attitude semantics*. This system provides a solution to the embedding problem and to the probability problem, while still accounting for the data that motivated the Box View. In more detail, the proposal (i) predicts that the acceptance conditions for a factual conditional $p \Rightarrow q$ accord with the Box View: $p \Rightarrow q$ is fully accepted iff all the epistemically possible p-worlds are q-worlds; (ii) makes $p \Rightarrow q$ and $p \Rightarrow \Box q$ logically equivalent, and $p \Rightarrow q$ and $\Diamond(p \wedge \neg q)$ logically inconsistent; (iii) explains why $p \Rightarrow q$ fails to contribute a universal quantifier when embedded under epistemic modals; (iv) given an epistemic modal M, it predicts the commutation $\mathsf{M}(p \Rightarrow q) \equiv p \Rightarrow \mathsf{M}q$; (v) in combination with a natural characterization of probability, it vindicates Adams' thesis, predicting that the probability of $p \Rightarrow q$ is the conditional probability of q given p.

In order to achieve these results, we adopted a novel semantic architecture: sentences are interpreted relative to two semantic parameters: an information state s, and an attitude A. Epistemic modals are treated as attitude shifters. This reflects the idea that epistemic modals like 'might', 'must', and 'probably', at least in some of their uses, do not contribute to the truth conditions of the sentence but are, instead, ways of expressing an attitude towards the prejacent. This idea is related to a tradition that views epistemic modals as *force modifiers* (see, among others Bybee, Perkins, & Pagliuca 1994; Huddleston & Pullum 2002; Price 1983; Schnieder 2010), whose role is to modulate the "force" with which a proposition is put forward. To see how this idea is vindicated in our setting, consider for example an assertion of "probably p" ($\mathsf{P}p$). With such an assertion, one is proposing to adopt a state that fully accepts this sentence. However, to fully accept $\mathsf{P}p$ is not to fully accept a proposition; rather, it is to bear a certain attitude to the proposition $|p|$, namely, partial acceptance to a high degree. Thus, when we compare an assertion of p with an assertion of "probably p", we see that the speaker is putting on the table each time the same proposition $|p|$, but recommending a different attitude towards it: full acceptance in one case, partial acceptance in the other. At the same time, we see that in our proposal, epistemic modals do make a compositional semantic contribution: this is crucial to accounting for their role in embedded contexts, as we illustrated in Section 5.1. Thus, AS vindicates some intuitions that motivated the force modifier view, while avoiding the main criticism usually moved to the view—namely, that it leaves us unequipped to account for epistemic modals embedded in truth-conditional constructions (von Fintel & Gillies 2007; MacFarlane 2011).

These merits are shared by some previous expressivist accounts of epistemic modals, in particular the one of Yalcin (2007; 2011). However, in Yalcin's account, modals are treated as contributing a quantification over epistemic alternatives; by contrast, in AS the role of epistemic operators is to shift a semantic parameter of the evaluation. This parameter, in turn, contributes the relevant quantification, but that happens only once a factual formula is eventually interpreted. The difference is immaterial when we look at plain epistemic statements like $\Diamond p$, since then the shift of the relevant parameter and the transformation of this parameter into a quantifier occur directly in a sequence, producing the same overall effect as if $\Diamond$ had directly introduced the quantifier. But the difference becomes crucial when we look at epistemic operators scoping above conditionals, as in $\Diamond(p \Rightarrow q)$: in this case, first $\Diamond$ shifts the attitude parameter to $\exists$; then the conditional updates the information state parameter from s to $s[p]$; and only at this stage, when it comes to assessing q, the attitude parameter contributes an existential quantification over the worlds in $s[p]$. Thus, the relevant quantification only shows up when we finally assess the consequent, after the restriction has taken place.

Turning from modals to conditionals, AS can be seen as providing a reconciliation of two different traditions, the Box View tradition and the probabilistic tradition (Adams 1965; 1975; Bennett 2003; Edgington 1986; 1995), which focused on somewhat different aspects of the semantics to conditionals. In AS, both can be recovered: the semantics of $p \Rightarrow q$ as given by AS predicts at the same time that (i) the conditional is logically incompatible with $\Diamond(p \wedge \neg q)$, and that (ii) its probability is the conditional probability of q given p. This shows that the insights from the two traditions need not lead to conflicting views about the semantics of conditionals, but can be seen as two different facets of a single semantics.

To conclude, let me mention three directions for further work. First, it would be interesting to study in more detail the formal and logical features of AS, especially since such an investigation would give us more insight into the repercussions of having an extra attitude parameter in the semantics.

Second, as we know from Lewis (1975), if-clauses can be used to restrict not only the domain of quantification of modals, but also that of other operators, in particular, adverbs such as *sometimes, usually*, and *always*, as in (27):

(27) Usually, if a man buys a horse, he pays cash.

It would be interesting to explore to what extent the proposal made here can be extended to deal with the interaction between conditionals and such adverbs, which seems to present many of the same puzzles that we discussed above. It seems that the same basic semantic architecture can be used, but the fundamental ingredients of the semantics must be construed somewhat differently: the analogue of the attitude parameter in these sentences would be a "frequency parameter", which the adverb shifts, and the analogue of an information state would be a domain of occasions/situations which the conditional restricts.[34] The fact that conditionals are treated as mere restricting devices is crucial in this setting as well. Indeed, Gillies (2010) gives a Box View account of conditionals which he argues to share the benefits of the restrictor theory in accounting for the way in which if-clauses restrict epistemic modals. However, Khoo (2011) shows that this account does not extend to adverbs of quantification: the problem, as Khoo clearly shows, is precisely that in order to get the right predictions we need to get rid of the universal quantification introduced by the conditional, and it is not clear how to do that. In the present approach, this problem does not arise, since conditionals do not introduce a universal quantification to begin with.

Finally, the proposal should be extended to cover subjunctive conditionals. These conditionals raise problems analogous to those discussed here for indicatives. For instance, (28-a) and (28-b) seem to mean the same:

(28) a. It is probable that if Alice had left she would have gone to London.
b. If Alice had left, it is probable that she would have gone to London.

The solution presented in this paper extendeds straightforwardly to this case. The crucial difference with the indicative case is that, for subjunctive conditionals, the hypothetical state resulting from an assumption is not obtained just by conditionalization, but must be computed by a different procedure, perhaps involving causal reasoning or a similarity ordering. The specifics of this procedure, however, do not matter to our solution of the embedding problem. Suppose $\Rrightarrow$ stands for the subjunctive conditional and $s\langle\alpha\rangle$ for the result of supposing α as a subjunctive assumption in state s. Suppose the semantics of $\Rrightarrow$ is given by a Ramsey-test clause analogous to the one for $\Rightarrow$:

34. This is not to say that sentences involving adverbial quantifiers don't express propositions: they do. The idea is the following: a basic sentence p (say "Alice has eggs for breakfast") is true relative to a world w and a specific occasion o (in our example, think of the occasion as a particular day). We can then ask whether a world w satifies p relative to a certain frequency (*universal*, if p is true at all the relevant occasions; *habitual*, if p is true at most occasions, etc.). The role of adverbial quantifiers is to shift the relevant frequency: for instance, "usually p" will be satisfied at a world w relative to an arbitrary frequency if p is satisfied at w with habitual frequency. We can then take the proposition $|\varphi|$ associated with a sentence to be the set of possible worlds where φ is satisfied with universal frequency. Then, for instance, the proposition expressed by "Alice usually has eggs for breakfast" will be the set of those worlds where on most days, Alice has eggs for breakfast.

$$s \vDash_A \alpha \Rrightarrow \varphi \qquad \Leftrightarrow \qquad s\langle\alpha\rangle \vDash_A \varphi$$

Then we have:

$$\begin{aligned} s \vDash_A \mathsf{P}(p \Rrightarrow q) \quad &\Leftrightarrow \quad s \vDash_{\pi_t} p \Rrightarrow q \\ &\Leftrightarrow \quad s\langle p\rangle \vDash_{\pi_t} q \\ &\Leftrightarrow \quad s\langle p\rangle \vDash_A \mathsf{P}q \\ &\Leftrightarrow \quad s \vDash_A p \Rrightarrow \mathsf{P}q \end{aligned}$$

Thus, the commutation of P and $\Rrightarrow$ is predicted.

Moreover, consider the probability issue. It seems that the probability of a subjunctive conditional $p \Rrightarrow q$ is just the probability that q is true in the state $s\langle p\rangle$ resulting from the counterfactual assumption of p.[35] This can be predicted in the same way as we did for epistemic conditionals in Section 4.3. Indeed, we have $s \vDash_{\pi_x} p \Rrightarrow q \;\Leftrightarrow\; s\langle p\rangle \vDash_{\pi_x} q \;\Leftrightarrow\; s\langle p\rangle(|q|) \geq x$. Since $\mathbb{P}_s(p \Rrightarrow q)$ is defined as the largest x such that $s \vDash_{\pi_x} p \Rrightarrow q$, we have that $\mathbb{P}_s(p \Rrightarrow q) = s\langle p\rangle(|q|)$, as expected.

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References

Adams, Ernest (1965). The Logic of Conditionals. *Inquiry*, *8*(1-4), 166–97.

Adams, Ernest (1975). *The Logic of Conditionals: An Application of Probability to Deductive Logic* (Vol. 86). Springer Science & Business Media.

Anand, Pranav and Valentine Hacquard (2014). Factivity, Belief and Discourse. In Luka Crnič and Uli Sauerland (Eds.), *The Art and Craft of Semantics: A Festschrift for Irene Heim* (69–90). MITWPL.

Belnap, Nuel (1970). Conditional Assertion and Restricted Quantification. *Noûs*, *4*(1), 1–12.

35. See, among others, Williams (2012), Leitgeb (2012), Schulz (2017), Schwarz (2018), Santorio (2018) for discussion of this thesis and the associated theoretical puzzles.

Belnap, Nuel, Michael Perloff, and Ming Xu (2001). *Facing the Future: Agents and Choices in Our Indeterminist World*. Oxford University Press.
Bennett, Jonathan Francis (2003). *A Philosophical Guide to Conditionals*. Oxford University Press.
Bledin, Justin (2014). Logic Informed. *Mind*, *123*(490), 277–316.
Bledin, Justin (2020). Fatalism and the Logic of Unconditionals. *Noûs*, *54*(1), 126–161. https://doi.org/10.1111/nous.12257
Bradley, Richard (2006). Adams Conditionals and Non-Monotonic Probabilities. *Journal of Logic, Language and Information*, *15*(1–2), 65–81.
Bybee, Joan, Revere Dale Perkins, and William Pagliuca (1994). *The Evolution of Grammar: Tense, Aspect, and Modality in the Languages of the World* (Vol. 196). University of Chicago Press Chicago.
Cariani, Fabrizio and Paolo Santorio (2018). Will Done Better: Selection Semantics, Future Credence, and Indeterminacy. *Mind*, *127*(505), 129–165. https://doi.org/10.1093/mind/fzw004
Ciardelli, Ivano (in press). The Restrictor View, without Covert Modals. *Linguistics and Philosophy*.
DeRose, Keith (1994). Lewis on 'Might' and 'Would' Counterfactual Conditionals. *Canadian Journal of Philosophy*, *24*(3), 413–18.
Edgington, Dorothy (1986). Do Conditionals Have Truth Conditions? *Crítica: Revista Hispanoamericana de Filosofía*, *18*(52), 3–39.
Edgington, Dorothy (1995). On Conditionals. *Mind*, *104*(414), 235–329.
Edgington, Dorothy (2008). Counterfactuals. *Proceedings of the Aristotelian Society 108*(1), 1–21.
Edgington, Dorothy (2014). Indicative Conditionals. In Edward N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy* (Winter 2014 ed.). https://plato.stanford.edu/archives/fall2020/entries/conditionals/
Egré, Paul and Mikaël Cozic (2011). If-Clauses and Probability Operators. *Topoi*, *30*(17), 17–29.
Evans, Jonathan and David Over (2004). *If*. Oxford University Press.
Fine, Kit (2017). Truthmaker Semantics. In Bob Hale, Crispin Wright, and Alexander Miller (Eds.), *A Companion to the Philosophy of Language* (556–77).
John Wiley & Sons. von Fintel, Kai (1998). The Presupposition of Subjunctive Conditionals. In Uli Sauerland and Orin Percus (Eds.), *The Interpretive Tract* (29–44). MIT Working Papers in Linguistics, *25*.
von Fintel, Kai (1999). NPI Licensing, Strawson Entailment, and Context Dependency. *Journal of Semantics*, *16*(2), 97–148.
von Fintel, Kai and Anthony Gillies (2007). An Opinionated Guide to Epistemic Modality. *Oxford Studies in Epistemology*, *2*, 32–62.
von Fintel, Kai and Sabine Iatridou (2002). If and When *If*-Clauses Can Restrict Quantifiers. Unpublished Manuscript. http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.11.7629
Gillies, Anthony (2004). Epistemic Conditionals and Conditional Epistemics. *Noûs*, *38*(4), 585–616.
Gillies, Anthony (2009). On Truth-Conditions for If (but Not Quite Only If). *Philosophical Review*, *118*(3), 325–49.
Gillies, Anthony S. (2010). Iffiness. *Semantics and Pragmatics*, *3*(4), 1–42.
Gillies, Anthony S. (2018). Updating Data Semantics. *Mind*, *129*(513), 1–41.

Hawke, Peter and Shane Steinert-Threlkeld (2021). Semantic Expressivism for Epistemic Modals. *Linguistics and Philosophy*, *44*, 475–511.

Heim, Irene (1988). On the Projection Problem for Presuppositions. In M. Barlow, D. Flickinger, and M. Wescoat (Eds.), *Proceedings of the Second Annual West Coast Conference on Formal Linguistics* (397–405). Stanford University Press.

Heim, Irene (1992). Presupposition Projection and the Semantics of Attitude Verbs. *Journal of Semantics*, *9*(3), 183–221. https://doi.org/10.1093/jos/9.3.183

Higginbotham, James (1986). Linguistic Theory and Davidson's Program in Semantics. In E. Lepore (Ed.), *Truth and Interpretation: Perspectives on the Philosophy of Donald Davidson* (29–48). Basil Blackwell.

Huddleston, Rodney and Geoffrey K. Pullum (2002). *The Cambridge Grammar of English*. Cambridge University Press.

Karttunen, Lauri (1974). Presupposition and Linguistic Context. *Theoretical Linguistics*, *1*(1–3), 181–94. https://doi.org/10.1515/thli.1974.1.1-3.181

Katzir, Roni and Raj Singh (2013). Hurford Disjunctions: Embedded Exhaustification and Structural Economy. In U. Etxeberria, A. Fălăuş, A. Irurtzun, and B. Leferman (Eds.), *Proceedings of Sinn und Bedeutung 18* (201–216)

Kaufmann, Stefan (2004). Conditioning against the Grain. *Journal of Philosophical Logic*, *33*(6), 583–606. https://doi.org/10.1023/B:LOGI.0000046142.51136.bf

Khoo, Justin (2011). Operators or Restrictors? A Reply to Gillies. *Semantics and Pragmatics*, *4*(4), 1–25. https://doi.org/10.3765/sp.4.4

Khoo, Justin and Paolo Santorio (2018). Probability of Conditionals in Modal Semantics. Lecture Notes for a course at NASSLLI 2018, available at https://justindkhoo.files.wordpress.com/2020/06/ks-nasslli2018.pdf

Klinedinst, Nathan (2010). Quantified Conditionals and Conditional Excluded Middle. *Journal of Semantics*, *28*(1), 149–70.

Kolodny, Niko and John MacFarlane (2010). Ifs and Oughts. *The Journal of Philosophy*, *107*(3), 115–43.

Kratzer, Angelika (1981). The Notional Category of Modality. In H. J. Eikmeyer and H. Rieser (Eds.), *Words, Worlds, and Contexts: New Approaches in Word Semantics* (38–74). Walter de Gruyter.

Kratzer, Angelika (1986). Conditionals. *Chicago Linguistics Society*, *22*(2), 1–15.

Leitgeb, Hannes (2012). A Probabilistic Semantics for Counterfactuals. Part B. *The Review of Symbolic Logic*, *5*(1), 85–121. https://doi.org/10.1017/S1755020311000165

Lewis, David (1975). Adverbs of Quantification. In Ed Keenan (Ed.), *Semantics of Natural Language* (3–15). Cambridge University Press.

Lewis, David (1976). Probabilities of Conditionals and Conditional Probabilities. In William L. Harper, Robert Stalnaker, and Glenn Pearce (Eds.), *Ifs* (129–47). Springer.

MacFarlane, John (2011). Epistemic Modals Are Assessment-Sensitive. In Brian Weatherson and Andy Egan (Eds.), *Epistemic Modality* (144–78). Oxford University Press.

Mandelkern, Matthew (2018). Talking about Worlds. *Philosophical Perspectives*, *32*(1), 298–325. https://doi.org/10.1111/phpe.12112

Meyer, Marie-Christine (2013). *Ignorance and Grammar* Doctoral dissertation, MIT. https://dspace.mit.edu/handle/1721.1/84420

Moss, Sarah (2013). Subjunctive Credences and Semantic Humility. *Philosophy and Phenomenological Research*, *87*(2), 251–78.

Moss, Sarah (2015). On the Semantics and Pragmatics of Epistemic Vocabulary. *Semantics and Pragmatics*, *8*(5), 1–81.

Popper, Karl (1959). *The Logic of Scientific Discovery*. Basic Books.
Price, Huw (1983). Does 'Probably' Modify Sense? *Australasian Journal of Philosophy*, *61*(4), 396–408.
Punčochář, Vít and Christopher Gauker (2020). Indicative Conditionals in Objective Contexts. *Theoria*, *86*(5), 651–87.
Ramsey, Frank Plumpton (1929). General Propositions and Causality. In Hugh Mellor (Ed.), *F. Ramsey, Philosophical Papers* (145–63). Cambridge University Press.
Santorio, Paolo (2017). Conditional Excluded Middle in Informational Semantics. In Thom van Gessel, Alexandre Cremers, and Floris Roelofsen (Eds.), *Proceedings of the 21st Amsterdam Colloquium* (385–94). https://hdl.handle.net/11245.1/12f840d2-c495-42fa-91be-deb9ef3c3b79
Santorio, Paolo (2018). Alternatives and Truthmakers in Conditional Semantics. *The Journal of Philosophy*, *115*(10), 513–49.
Schnieder, Benjamin (2010). Expressivism Concerning Epistemic Modals. *The Philosophical Quarterly*, *60*(240), 601–15.
Schulz, Moritz (2014). Counterfactuals and Arbitrariness. *Mind*, *123*(492), 1021–1055.
Schulz, Moritz (2017). *Counterfactuals and Probability*. Oxford University Press.
Schwarz, Wolfgang (2018). Subjunctive Conditional Probability. *Journal of Philosophical Logic*, *47*(1), 47–66.
Stalnaker, Robert (1978). Assertion. In Peter Cole (Ed.), *Pragmatics*, 315–32. Brill.
Starr, William (2014a). A Uniform Theory of Conditionals. *Journal of Philosophical Logic*, *43*(6), 1019–64.
Starr, William (2014b). Indicative Conditionals, Strictly. Manuscript, Cornell University. http://williamstarr.net/research/indicative_conditionals_strictly.pdf
Veltman, Frank (1981). Data Semantics. In Jeroen Groenendijk, Theo Janssen, and Martin Stokhof (Eds.), *Formal Methods in the Study of Language* (43–63). Mathematical Centre.
Veltman, Frank (1985). *Logics for Conditionals*. Doctoral dissertation, University of Amsterdam. https://www.illc.uva.nl/Research/Publications/Dissertations/HDS-02-Frank-Veltman.text.pdf
Veltman, Frank (1996). Defaults in Update Semantics. *Journal of Philosophical Logic*, *25*(3), 221–61.
Willer, Malte (2013). Dynamics of Epistemic Modality. *Philosophical Review*, *122*(1), 45–92.
Willer, Malte (2014). Dynamic Thoughts on Ifs and Oughts. *Philosophers' Imprint*, *14*(28), 1–30.
Willer, Malte (2018). Simplifying with Free Choice. *Topoi*, *37*(3), 379–92.
Williams, Robert (2012). Counterfactual Triviality: A Lewis-Impossibility Argument for Counterfactuals. *Philosophy and Phenomenological Research*, *85*(3), 648–70. https://doi.org/10.1111/j.1933-1592.2012.00636.x
Yalcin, Seth (2007). Epistemic Modals. *Mind*, *116*(464), 983–1026.
Yalcin, Seth (2011). Nonfactualism about Epistemic Modality. In Andy Egan and B. Weatherson (Eds.), *Epistemic Modality* (295–332). Oxford University Press.
Yalcin, Seth (2018). Conditional Belief and Conditional Assertion. Handout for a talk given at LMU Munich, December 7th 2018.