There is No Anima Mundi: Leibniz on the Impossibility of a Soul of the World

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The main source of perplexity raised by Leibniz's (mathematical) argument against the soul of the world stems from the idea that it is the infinity of the universe that precludes it from having a soul. But if this is so, how is it possible that organic bodies, which, having infinitely many parts, are also infinite, are endowed with a soul? The present paper aims to provide a new solution to this puzzle. The solution explains the difference between the body and the universe by looking at how their respective parts are arranged. It is the arrangement of the parts of the body that allows a body to be divided into infinitely many parts whilst, at the same time, having a finite magnitude. By contrast, the way in which the alleged parts of the world are arranged makes it impossible that the world has a finite magnitude: the world cannot be a whole, and so it cannot have a soul. The consequence is that it does not matter *how many* parts bodies have, but only that they have a finite *magnitude*. In this case, bodies respect the Part-Whole Principle (the whole is bigger than any of its proper parts) and therefore can be described as finite wholes with parts.

1. Introduction

This paper focuses on Leibniz's (mathematical) argument against the existence of a soul of the world. Leibniz's argument relies on a rejection of infinite numbers and infinite wholes to argue for the impossibility of a world soul. Recently this argument has gained a lot of scholarly attention. The main source of perplexity stems from the central idea on which the argument is based— that it is the infinity of the universe that *precludes* it from being a whole, and consequently from having a soul; together with Leibniz's contention that organic bodies, which are

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composed of infinitely many parts (and hence are also infinite), *are* endowed with souls.

This problem has proven harder to deal with than what one might at first think. Indeed, scholars have divided themselves between different approaches in attempting to provide a solution. The three main proposals are those of Carlin (1997), then defended and developed by Arthur (1999; 2001), Brown (1998; 2000; and especially 2005), and Harmer (2014). In the present paper, my aim is to provide a new solution to the puzzle. This solution explains the difference between the body and the universe by looking at how their respective parts are arranged. It is the arrangement of the parts of the body that allows a body to be divided into infinitely many parts whilst, at the same time, having a finite magnitude. The body can thus be considered a whole and can thus have a soul. By contrast, the way in which the alleged parts of the world are arranged makes it impossible that the world has a finite magnitude: the world cannot be a whole, and so it cannot have a soul.

I will develop and clarify the solution just sketched as follows. In §2 I introduce Leibniz's position on the soul of the world, explaining why, for him, it was so important to reject the world soul. In §3 I reconstruct Leibniz's argument as a *modus tollens*, with the following two premises: "if the world has a soul, it is a whole (with parts)," and "the world is not a whole." Section 4 focuses on the second premise: here a key role is played by Leibniz's theory of measurement (or estimation) of quantity. In particular, such a theory is used to clarify in which sense bodies can be considered wholes, while the world cannot. Section 5 deals with the first premise and explains the connection between "having a soul" and "being a whole." Such an explanation is based on the result of the analysis developed in §4: if the world *has* a soul, then it is a body of a living organism, and since it is a body, it is a whole (with parts). In this way, our defense of the fact that bodies can be considered as wholes with parts proves immediately fruitful by providing us with a clear explanation of premise 1 that is otherwise unavailable. Moreover, I argue that the claim "if the world *has* a soul, then it is a body of a living organism" is a direct consequence of Leibniz's conception of the world soul as something immanent (rather than transcendent) to things. Section 6 focuses on what the argument can teach us with respect to two important aspects of Leibniz's thought, namely his rejection of infinite number, and the role that the notion of quantity plays in his metaphysics. Finally, §7 compares my interpretation with those in the existing literature, delineating what of them I accept and reject.

Even though I believe that the different proposals that already exist in the literature have, with different degrees of success, partially unraveled the problem addressed here, it seems to me that none of them are without fault. The major upshot of their shortcomings is to hide the important lesson that the argument

can give us on the role that the notion of quantity plays in Leibniz's metaphysics. It is therefore necessary to go back to this problem once again.

2. What is at Stake with the World Soul

Leibniz first rejected the claim that the world has a soul in April 1676.[1] This rejection appears in a number of texts spanning his whole life, including the correspondence with Samuel Clarke, where Leibniz charges Clarke and the Newtonians with being committed to the idea that God is the soul of the world as a consequence of the view that space is the *sensorium* (i.e. the organ) through which God perceives the world. Indeed, Leibniz seems to use this accusation as a *reductio* argument against this view.[2]

Leibniz regarded the idea that the world has a soul as equivalent to the claim that the world is a living organism.[3] In *Considérations sur la doctrine d'un esprit universel unique* (GP VI 529–538), written in 1702, Leibniz explains that this idea can be constructed in two ways: the first is to identify this living organism with God himself, which amounts to an identification of the soul of the world with God; the second avoids this identification and maintains that while God transcends the world (he is an *intelligentia supramundana*, as Leibniz tells Clarke[4]), nevertheless the world has a soul (which would therefore be a created entity). For the former position, Leibniz had in mind Spinoza, for whom God is Nature (Spinoza is mentioned a few lines before where Leibniz writes that "Spinoza, who admitted only one substance, was not that far away from the doctrine of the universal spirit … ." [GP VI 530]), but also some Platonists and the Stoics: "I would have easily dared believe [*facile crediderim*] that many oriental philosophers, no less than the Platonists and the Stoics, held God to be the soul of the world or a universal nature immanent to the things" (GP II 383).[5] As argued by Kulstad (2008: 93–95), the main problem of identifying God with the soul of the world is the denial of his transcendence: the *anima mundi* is simply an *intelligentia*

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1. See Carlin (1997: 1–4).

2. In the correspondence with Clarke, the nonexistence of the soul of the world is cited in the second, fourth, and fifth letters. In the latter, §82, Leibniz writes: "If it is by means of a sensorium that God perceives what passes in the world, it seems that things act on him and that therefore he is what we mean by a soul of the world. The author charges me with repeating objections without taking notice of the answer, but I do not see that he has answered this difficulty" (LC 55).

3. This is clear from the passage of the *Theodicy* (§195) that we quote below. See also §5 below.

4. Second letter, §10 (LC 10).

5. Leibniz uses the potential subjunctive at the first singular person (*facile crediderim*) which expresses a caution or modest assertion, as stressing that it does not matter whether these philosophers explicitly endorsed such a view, but only that he (Leibniz) believed that such a view follows from their premises.

mundana.[6] Moreover, this seems easily to lead to a view where the divinity acts not in virtue of a rational choice, but in virtue of a brute necessity: "those who … explain everything mechanically by means of a certain blind force of nature … take God away from us or they transform God into a certain brute force of general nature, robbed of intellect and will, that is to say, they transform God into the soul of the world" (A VI 4, 460–461, trans. from Kulstad 2008: 95).

Leibniz therefore had good reasons to reject the view that God is the soul of the world. But what about the second position which holds that the *anima mundi* is a created entity? In the *Considérations*, this position is identified with that of the Cabalists, Henry More and "certain chemists, who have believed that there is a universal arché or a soul of the world …".[7] Clearly, we cannot appeal to a certain conception of God to dismiss this position. However, Leibniz needs to reject even this position to safeguard the internal coherence of his own metaphysics. Leibniz holds two *apparently* inconsistent theses: the presence of the actual infinite, especially in reality (as an expression of God's power), and the rejection of any categorematic infinity, such as infinite wholes and numbers. The apparent contradiction is resolved by reading the actual infinite in a syncategorematic or distributive way: there are actually infinitely many things which are more than can be counted or expressed by any (finite) number.[8] But if the world were a

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6. This shows that, strictly speaking, Leibniz does not need the "mathematical" argument to reject the idea that God is the soul of the world, since he can appeal to the fact that God transcends the world, while the soul does not transcend its own body. Or, as he suggests in GP II 304–305, another reason why God cannot be the soul of the world is that God is the cause of the world, while the soul is not the cause of its own body.

7. Here is the full paragraph: "The doctrine of the universal spirit is good in itself, because everyone who teaches it, in effect, admits the existence of the divinity, both those who believe that this universal spirit is supreme, and so then they believe that it is God himself, and those who believe with the Cabalists that God has created it, which was the opinion of the English Henry More as well, of those other new philosophers and in particular of certain chemists who had believed that there is a universal arché or soul of the world, and some of them have claimed that this is the spirit of the Lord that moves on the water, of which the beginning of Genesis speaks" (GP VI 530–31, my translation). A few comments on this passage are in order: 1. Leibniz talks here of the universal spirit, not only of the soul of the world. He presents different interpretations of this concept of which the soul of the world is one; 2. This is the only mature text (after 1676) that I know of where Leibniz seems to entertain a positive attitude towards the soul of the world: however, far from endorsing it, he simply stresses that a positive element of the doctrine consists in the fact that it implies a recognition of the existence of divinity. This seems to me only an instance of Leibniz's attitude towards finding something valuable in views otherwise very different from his own; 3. For the previous reason, I do not agree with Vassányi (2014: ch. 3, §5) who argues that, in this text, "Leibniz can now reasonably say in respect of the *world soul conceived as subordinate to God* that it is philosophically tolerable but not necessary. At least such a relatively complex philosophical attitude is expressed by his refusal (in *De ipsa natura*) and acceptance (in *Considérations sur la doctrine d'un esprit universel unique*) of this doctrine" (Vassányi 2014: 61).

8. The distinction between categorematic and syncategorematic infinite is a distinction on the scope of the quantifiers: x is a categorematic infinite if x is bigger than any finite magnitude y; by contrast, x is a syncategorematic infinite if, for any finite magnitude y, x is bigger than y. For

living organism, because of its infinity, it would exactly be an instantiation of the categorematic infinite. The rejection of the soul of the world, independently of whether it is identified with God, is thus a necessary step to take in order to maintain the internal coherence of the position; once the infinity of the universe is admitted, it is not possible to hold that there is a soul of the world and at the same time to reject the notion of categorematic infinite.

Therefore, by rejecting the idea of the world soul, Leibniz is rejecting all these different scenarios at once; that is, *in one single move* he was able to dismiss a wide array of rival positions. Moreover, the mathematical nature of his (main) argument made this rejection – at least to Leibniz's eyes – not a contingent one, but a necessary one.

3. The Argument

Leibniz adopted different argumentative strategies against the world soul in different texts. For example, in April 1676 (Leibniz's first rejection of this notion), he argues that there is no soul of the universe, because the latter is an entity by aggregation, and no soul can be an entity by aggregation.[9] Clearly, such an argument could not withstand the development of Leibniz's thought: in his mature metaphysics, each living being has a soul despite having a body which is an entity by aggregation. In *De ipsa natura* of 1698, Leibniz claims that the hypothesis that there is a soul of the world is superfluous. Here the context is that of natural philosophy, and Leibniz is defending the view that there is an active force in each living being, bestowed upon them by God. The idea is that, once we recognize such a force, the traditional reasons that have led thinkers to postulate the world soul fail.[10] However, these arguments are only sketched, and

example, an infinite collection or an infinite whole with an infinite number of terms/parts is a categorematic infinite. By contrast, if we consider infinitely many terms such that, given any finite number of them, more can be found (but for which there is no infinite number of them), then we have a syncategorematic infinite. The distinction has often and erroneously been confused with the distinction between actual and potential infinite. However, an actual infinite (an infinite where each of its terms is given) can be interpreted categorematically or syncategorematically. On the distinction categorematic/syncategorematic infinite see Uckelman (2015). On the syncategorematic actual infinite in Leibniz, see Arthur (2018a).

9. "There is no soul of the world, because the continuum cannot be composed of minds, as it can be composed of spaces. You will say that such a soul does exist in a certain way, in so far as minds sense themselves. I say in reply that a soul cannot be an entity by aggregation, but that universal space is an entity by aggregation. So it is not surprising that there is no soul of the universe" (A VI 3, 521 / DSR 81).

10. "… I certainly agree that there is no such thing as the soul of the universe. I also agree that those wonders which present themselves daily, and about which we customarily say (quite rightly) that the work of nature is the work of intelligence, should not be ascribed to certain created intelligences endowed with wisdom and power [virtus] only in proportion to the task at hand, but

cannot stand the comparison with the main "mathematical" argument against the soul of the world. The latter is repeated in at least four different texts, and in one of them it is even presented as a *demonstration* of the nonexistence of such an entity: "that God is not the soul of the world can be *demonstrated…*" (A VI 4, 1492, emphasis added). It is this argument that we will analyze in the rest of the paper.[11]

In the first half of the 1680s, Leibniz presents the argument against the soul of the world on different occasions. For example, in *De mundo praesenti,* he writes that

> The aggregate of all bodies is called the world, which, if it is infinite, is not even one entity, any more than an infinite straight line or the greatest number are. So God cannot be understood as the World Soul: not the soul of a finite world because God himself is infinite, and not of an infinite world because an infinite body cannot be understood as one entity (*unum Ens*), but that which is not one in itself (*unum per se*) has no substantial form, and therefore no soul. So Marianus Capella is right to call God an extramundane intelligence. (*De mundo praesenti*, March 1684 - Spring 1686: A VI 4, 1509)[12]

As the first line makes clear, the argument is dilemmatic: either the world is finite or infinite. Indeed, Leibniz argues that the world is not one entity, based only on the supposition that it is infinite. Similarly, in *Deum non esse mundi animam* (A VI 4, 1492), the argument is built as a dilemma: either the world is finite, and then God cannot be the world soul because he is infinite, or the world is infinite, and so it is not one entity.[13] However, the dilemmatic form disappears

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rather that the whole of nature is, so to speak, the workmanship of God … . And so, I think that the omniscient heat of Hippocrates, and Avicenna's Cholcodean giver of souls, the exceedingly wise plastic virtue of Scaliger and others, and the hylarchic principle of Henry More are in part impossible, and in part unnecessary" (AG 156).

11. For an analysis of the other argumentative strategies against the *anima mundi*, see Kulstad (2008: §4) and Vassányi (2014: ch. 3).

12. Other texts belonging to the same period are *Deum non esse mundi animam* (A VI 4, 1492) and *De infinito et immensum.* In the latter, the world soul is not mentioned, but Leibniz makes the main claim at the heart of the argument, i.e. that the world is not one or a whole. The text is quoted in §4.4 below.

13. "That God is not the soul of the world can be demonstrated, for the world is either finite or infinite. If the world is finite, then God, who is infinite, certainly cannot be said to be the soul of the world; but if it is assumed that the world is infinite, it is not one being, i.e. a body one in itself (*unum per se corpus*) (just as it has previously been demonstrated that the infinite in number and magnitude is neither one nor whole; but that only the infinite in perfection is one and whole). Therefore, no soul can be understood as belonging to it. Certainly an infinite world is no more one and whole than infinite number, which Galileo demonstrated is neither one nor whole" (A VI 4, 1492, English translation from Brown 2005: 450).

in later writings when Leibniz introduces the argument by stating that the world is infinite.[14] An example is the *Theodicy*:

> There is an infinity of creatures in the smallest particle of matter, because of the actual division of the continuum to infinity. And infinity, that is to say the accumulation of an infinite number of substances, is, properly speaking, not a whole any more than infinite number itself, whereof one cannot say whether it is even or uneven. That is just what serves to confute those who make of the world a God, or who think of God as the Soul of the world; for the world or the universe cannot be regarded as an animal or substance. (G VI 232 / T §195)[15]

What seems clear is that the argument runs in parallel to the argument against infinite number. The same reason that prevents the existence of an infinite number prevents the existence of the soul of the world. This reason is the violation of the Part-Whole Principle (i.e. the idea that the whole is always strictly bigger than each of its parts), on the supposition that an infinite whole exists.[16] Since numbers (in particular, positive integers) are for Leibniz wholes or collections made of (mathematical, i.e. abstract) unities, the supposition of the existence of an infinite number amounts to the supposition of the existence of an infinite whole of unities (where each unit is a part of this whole). But if we remove from this whole a finite number of unities, the remainder will still be infinite, i.e. it will be equal to the original whole. As a consequence, a (proper) part of the whole would be *equal* to the whole, which is impossible in virtue of the Part-Whole Principle. Therefore, there is no infinite whole of unities, and so no infinite number.[17]

Similarly, in the case of the world soul, the supposition of its existence requires that the universe is an infinite whole ("the accumulation of an infinite number of substances"), which contradicts the Part-Whole Principle. The argument exploits the concepts of part and whole. Indeed, the part-whole relation can be considered to be *a unique relation*: if something has parts, then it is a whole,

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14. This is hardly a substantial change, since Leibniz had adopted the view that the world is infinite already at the end of the 1670s: see, for example, (A VI 4, 1393 / LLC 235). It seems to me that the dilemmatic form that the argument has in the 1680s is due to the desire to provide an argument which could be effective even for those who did not accept the infinity of the world.

15. Another late text where the argument is presented is a letter to Des Bosses dated 11 March 1706 (GP II 304–305). We analyze it in §5.

16. Leibniz considers what I am calling the Part-Whole Principle a necessary truth which can be proved by analyzing the definitions involved. For a proof of it, see for example GM VII 20.

17. The standard version of this argument exploits Galileo's Paradox according to which the square numbers are less than the positive integers since the former are a part of the latter, and at the same time they are as many as the integers since there is a bijection between the two collections. See for example (A VI 3, 168); on Leibniz's rejection of infinite number see also Van Atten (2011), Esquisabel & Quintano (2017), and Costantini (2020).

and something is a whole only if it has parts: "it is also clear that a part inheres in the whole, or, in other words, that as soon as a whole is given, a part is immediately given as well …" (GM VII 274). In other words, to deny that something is a whole amounts to denying that we can apply to it the part-whole relation.[18]

But what exactly does the word 'world' (or 'universe') refer to? At stake here is the *created* world, i.e. the aggregate of all (created) substances, or as Leibniz explicitly puts it before introducing this very argument in *De mundi praesenti* (A VI 4, 1509), the aggregates of all (created) bodies. As such, God is excluded from the world.[19] At this point, we can reconstruct Leibniz's argument as the following *modus tollens*:

Premise 1: If the world has a soul, then it is a whole.
Premise 2: The world is not a whole.
Conclusion: Therefore, the world has no soul.

Premise 1 sounds quite enigmatic. I take it that it is clear from the passage of the *Theodicy* above that Leibniz endorses it.[20] Indeed, the key point of the argument is the claim that "infinity, that is to say the accumulation of an infinite number of substances, is, properly speaking, not a whole." This is followed by the further statement "that is just what serves to confute those who make of the world a God, or who think of God as the Soul of the world." It is therefore clear that the world has no soul because it is not a whole, which is what premise 1 claims; less clear is why Leibniz believed in its truth. I shall propose an interpretation of it in §5. Notice that the formulation of this premise makes the attribute of 'being a whole' a necessary condition for having a soul.[21]

Premise 2 is less enigmatic, and constitutes the mathematical side of the argument, from which I shall start my analysis.

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18. That the part-whole relation is a unique relation is a subtle but important point that is worth stressing again. In fact, many commentators seem to have missed it. Russell (1937: 115–116) writes that "Leibniz's position is this: that a notion of a *whole* can only be applied to what is substantially indivisible. … One whole must be one substance … ." Here Russell is conflating the notion of whole with that of substance.

19. The claim that the world is not a whole does not contradict the thesis defended by Rutherford (2021) that the world is one precisely because it depends on a single cause (and can be grasped by God in a single idea). In §5 we argue that since the world is not a whole, it is not one. This amounts to denying that the world is a single object, with the consequence that the world is simply the multiplicity of all substances. This is fully consistent with the sense of "unity" in Rutherford's argument: the multiplicity of substances is one in the sense that they depend on a single cause and can be grasped by God in a single idea.

20. The same claim can be found in a letter to Des Bosses dated 11 March 1706 (GP II 304–5). The text is quoted and analyzed in §5.

21. Clearly, being a whole is not sufficient to have a soul. All finite geometrical objects (lines, figures, volumes etc.) are wholes but they do not have any soul.

4. Premise 2: the World is Not a Whole

Leibniz's defense of premise 2 is quite straightforward and can be summarized with the following argument:

2a. The world is an infinite aggregate of substances.
2b. No infinite aggregate is a whole.
Conclusion: Therefore, the world is not a whole.

Premise 2a simply states Leibniz's definition of the (created) world. The world is the aggregate of all created substances, but there are infinitely many substances, therefore the world is an infinite aggregate.[22]

The truth of Premise 2b follows from the Part-Whole Principle. Therefore, at the basis of premise 2 is Leibniz's solution to Galileo's paradox, namely the idea that the part-whole relation cannot be applied to infinite aggregates. An infinite aggregate is not a whole on pain of contradiction.[23] The world is an aggregate of substances, but these substances are not part of it, which implies that we cannot identify a part of the aggregate which is as big as the whole.[24]

Implicit here is a distinction between aggregates that can be unified by a soul and aggregates that cannot be unified by a soul. From a metaphysical point of view, both are pluralities, multitudes, and not substantial unities. But only the former can be treated as wholes and consequently they can be endowed with a soul (according to premise 1, which we discuss below). In other words, they can be bodies of something (i.e. a dominant monad) that has an enduring, and not merely accidental, unity. For this reason, we can call them "mere bodies" (i.e. bodies independently of their dominant monads). In this sense, the universe is not a mere body: it is not an aggregate that can be treated as a whole and thus can have a soul.

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22. See for example (A VI 4, 1393 / LLC 235), quoted in §4.2.

23. I am here distinguishing between mere aggregates and wholes with parts. Even though Leibniz is not always careful to distinguish the two notions, the distinction is fundamental, and it is grounded on the same definition of parthood as what is in (*inesse* relation) the whole and is homogeneous to it: "We say that an entity is in [*inesse*] some locus, or is an ingredient of something, if, when we posit the latter, we must also be understood, by this very fact and immediately, without the necessity of any inference, to have posited the entity as well. Thus when we posit any finite line, we also posit its end points as belonging to it. An entity which is in something and is also homogeneous to it is called a part, and that which it is in is called a whole; or a part is a homogeneous ingredient of a whole" (GM VII 19 / PPL 667–668). As I am using 'aggregate', it indicates something composite with regard to the *inesse* relation, not the parthood relation. Therefore, every whole is an aggregate (since parthood is a special case of *inesse*), but not all aggregates are whole (since *inesse* is a more comprehensive relation than parthood). In particular, no infinite aggregate is a whole.

24. I will not discuss the problem of the soundness of this argument. Clearly, Leibniz's rejection of a world soul stands or falls with his rejection of infinite number. On this topic, see the discussion in Arthur (1999; 2001) and Brown (1998; 2000).

Clearly this leaves totally unexplained the original concern: bodies too are infinite aggregates of other bodies but, contrary to what happens with the world, they have a soul. To unravel this problem, we should understand better what it means for an aggregate to be a whole with parts, and why Leibniz rejects infinite wholes. We shall argue that it does not matter *how many* parts bodies have, but only that they have a *finite magnitude*. A body can have infinitely many parts and a finite magnitude in virtue of the arrangement of its parts. By contrast, the way in which the alleged parts of the world are arranged makes it impossible that the world has a finite magnitude: the world cannot be a whole. It is the magnitude and not the number of parts that distinguishes bodies (aggregates that can be treated as wholes with parts), from the world (an aggregate that cannot be treated as a whole).

4.1 What it is Like to be a Whole

Leibniz's mereology, i.e. the theory of the parthood relation, is a restriction of the *inesse* relation via the relation of homogeneity.[25] To understand the role that mereology plays within Leibniz's mathematics and philosophy, it is enough to look at the notion of homogeneity. Homogeneity indicates quantitative comparability: if x and y are homogeneous, then there is a ratio between them, i.e. they can be exactly compared with regard to their quantity.[26] Mereology thus works as a backbone theory for Leibniz's theory of quantity. In different places, Leibniz characterizes quantity in the following way:

> We can define the quantity of a thing as the property of the whole insofar as it has all its parts (GM VII, 30 / A VI 4, 418).[27]

Such a characterization claims that "having a quantity" implies "being a whole with parts." This works as a kind of axiomatic principle that allows Leibniz to apply mereology in the determination or estimation of quantities. The converse is also valid: "being a whole" implies "having a quantity." To see this, just suppose that W is a whole with parts. By definition of parthood, each of its parts are homogeneous with W, which means that there is a ratio between W

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25. The *inesse* relation is the key relation of Leibniz's Real Addition Calculus. For the latter see Lenzen (1989) and Mugnai (2019). See also Costantini (2023) for a philosophical interpretation of the notion of aggregate that takes into consideration the Real Addition Calculus.

26. "There is ratio only between homogeneous quantities, and this is clear by definition" (GM VII 34). Or: "Among all the relations, the simplest is that which is called ratio or proportion, i.e. a relation between two homogeneous quantities, which originates only from them, without assuming a third [quantity]" (GM VII 23).

27. See Rescher (1955) for a deep analysis of the notion of quantity in Leibniz.

and each of its parts. So W has quantity.[28] We can thus summarize this principle in the following bi-conditional that I shall call Part-Quantity:

(Part-Quantity): *x has quantity if and only if x is a whole with parts*

Mereology is thus a theory whose main application is in mathematics; in particular, it plays a central role in the way in which quantities or magnitudes are determined.

4.2 *Bodies are Wholes*

Given Part-Quantity, the rejection of infinite wholes amounts to the rejection of infinite quantities or magnitudes. To claim that the world is not a whole (premise 2) amounts to claiming that the world has no quantity/magnitude. However, each body has a (finite) magnitude. Indeed, this is something widely recognized in the secondary literature. In particular, Carlin's and Arthur's claim that bodies have an *arithmetical unity* seems to amount to the fact that they have a finite quantity (a finite size, a finite volume, a finite weight, a finite amount of force, etc.).[29] But if bodies have quantities, by Part-Quantity it follows that they are wholes with parts. The argument is a simple *modus ponens:*

Premise A. Bodies have (finite) quantity/magnitude.
Premise B. If x has quantity, then x is a whole with parts (left-to-right direction of Part-Quantity).
Conclusion: Bodies are wholes with parts.

The conclusion is far from obvious: indeed, it has explicitly been rejected.[30] A natural objection to it amounts to noticing that Leibniz does not usually refer to bodies as wholes of parts. For example, the following is a recurrent characterization of bodies:

A body is not a substance but an aggregate of substances (A VI 4, 1668).

28. The characterization of quantity via the parthood relation is not new, but is already present in Aristotle (*Metaphysics* 5, 13).

29. Arthur (1999: 112) writes: "Here I think Carlin gave the right answer when, after quoting Leibniz to the effect that the world, being infinite in magnitude, 'would not be a single body, nor could it be regarded as an animal', he explained that, unlike finite bodies, 'the actual infinite [in magnitude] does not even possess what Leibniz would call arithmetical unity'."

30. See Harmer (2014: 254): "In virtue of having infinitely many parts, the body of a composite substance will be merely a multitude, not *one whole*. Similarly, it will not have a quantity, except fictionally … ." In fact, Harmer seems here to deny premise A of the argument.

In addition, in *Monadology* §2, what is composed (which includes bodies) is said to be an accumulation or aggregate of simples, and in the *Principle of Nature and Grace* §1 "the composed substances, i.e. the bodies, are multitudes." Leibniz usually talks of bodies in terms of aggregates, multitude, plurality of substances, not in terms of wholes. However, when he deals with the actual division of bodies, he explicitly says that the division produces *parts* and that the parts are further bodies:

> Created things are actually infinite. For any body whatever is actually divided into several parts, since any body whatever is acted upon by other bodies. And any part whatever of a body is a body by the very definition of body. So bodies are actually infinite, i.e. more bodies can be found than there are unities in any given number. (A VI 4, 1393 / LLC 235)

If we now keep in mind the above fact, i.e. that if something has parts, it is a whole, and that every whole has parts (i.e. there are no parts without wholes, and conversely), then it seems we have a problem: on the one hand, bodies are not usually referred to as wholes by Leibniz (and so if they are not wholes, they should not have parts); on the other hand, Leibniz explicitly uses the parthood relation when he speaks of the division of bodies by their inner motions (which would imply that bodies are wholes). Moreover, a part of a body is a further body: as the original body has a volume, a size, a magnitude, etc., so too does each of its parts. This clearly shows that a part of a body is *homogeneous* to the body itself. And since we are talking of things which *are in* (*inesse*) the body, the body should be a whole with parts (just in virtue of the definition of parthood).

The objection can be dismissed by interpreting the passages just quoted as expressing two different ways in which we can analyze bodies. On one side, a body is a portion of extended matter, a well-founded *phenomenon* divided into further bodies by its internal motions. This is the conception of a body in which we are mainly interested in whilst doing physics or other empirical sciences. It is with regard to this conception of bodies that we can apply the part-whole relation to them and say that bodies can be said to be wholes with parts. Indeed, the argument above was based on premise A, which regards the magnitude of bodies. On the other side, a body is an aggregate of substances. It is a *well-founded* phenomenon. These substances constitute bodies in the sense that the reality (or being) of bodies is grounded on the reality (or being) of substances. This is the conception of the body that we are interested in whilst doing metaphysics. Here, we should not apply the part-whole relation, but the *inesse* relation: substances are in bodies, but not parts of them. When we look at bodies from this metaphysical angle, we cannot say that a body is a whole whose parts are its

substances, otherwise we would have an infinite whole such as the whole of all unities, which—we know—contradicts the Part-Whole Principle.

To sum up, a body can be seen as a whole, despite having infinitely many sub-parts (or sub-bodies); but it cannot be seen as a whole whose parts are its own substances. The first sense is compatible with the Part-Whole Principle; the second sense is not. The reason is that the first sense admits a division into parts which are not, generally speaking, *disjoint*, while if substances were parts of bodies, they would be disjoint parts. To better understand this point, it is useful to look at how measurement of quantity works for Leibniz. Moreover, this explanation will make definitively clear why the world has no definite quantity, contrary to the body.

4.3 Leibniz's Method for Measuring Quantities

Leibniz's canonical method for measuring quantities consists in generalizing to any kind of magnitude (included incommensurable ones), the Euclidean algorithm to determine the greatest common divisor between two integers.[31] Suppose we want to compare a line-segment, A, with regard to a second line-segment, B, which is taken as the measure. The procedure works as follows: first, we should divide the longest segment (say A) into parts congruent to B. Division here is simply repeated subtraction: we start subtracting parts of A congruent to B until it is no longer possible. At this point, we take the remainder (which is possibly null) and we start subtracting the remainder from B, until it is no longer possible, and we keep on proceeding in this way. There are two possibilities: either at some point the procedure stops, which means that the last remainder is the greatest common measure between A and B, or the procedure never stops, which means that A and B are incommensurable. This division procedure gives us as a result a sequence of quotients (i.e. positive integers) that exactly determines the ratio between the two magnitudes, i.e. the quantity of one in terms of the other. This measurement procedure consists thus in the mereological subtraction of a part of the measured magnitude congruent to the chosen measure. When we subtract a part from a magnitude, what is left —the remainder—is not only strictly smaller than the original magnitude, but it is disjoint from the part subtracted. In other terms, the division of a magnitude is a division into *disjoint* parts (that Leibniz calls *partes extra partes*), i.e. parts that have no parts in common.

Equivalently, we may think of this procedure as the result of establishing how many times a quantity conceived as a measure is contained in the quan-

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31. See for example (A VI 4, 420) or (GM VII 23).

tity to be estimated (or how many times the measure must be repeated to comprise the measured object). This is a different perspective on the same procedure: again, consider the same line-segments A and B. The idea is to repeat the measure B as many times as needed to comprise the entire A. If – after *n* repetitions – we are left with a remainder smaller than the measure, we start repeating this remainder as many times as needed to comprise the measure B, and we repeat this operation iteratively, until the procedure eventually stops. The repetition of the measure amounts to considering *disjoint* copies of the measure and summing them together: the idea is that the repetition of the same measure corresponds to the mereological sum of disjoint objects B′, B″, B‴, etc., which are congruent with the measure.[32] Measurement consists in the repetition of disjoint parts.

Now that the measurement procedure for quantities have been introduced, we can apply it to determine the quantity of a body. To do that, we need to choose an object as our measure. Once the measure is fixed, we look at how many times the measure is contained in the body, i.e. we look at how many times we need to repeat the measure in order to comprise the entire body. This procedure requires us to abstract from many features of the body and the assumed measure, i.e. they must be considered as idealized entities. We won't be directly interested in the actual parts of the body, but only in the fact that we can ideally divide the body into parts congruent to our measure.[33]

Notwithstanding the need for idealization in applying the measurement procedure, how the actual parts of the body are arranged and structured with respect to one another still matters for the procedure to be applied. In other words, what allows us to measure A in virtue of its part B is the way in which the actual parts of a body are structured. Not all possible ways in which the parts are arranged makes measurement possible. To better understand this point, we must look at how a body is divided into actual parts.

> In every body there is some force or motion. No body is so very small that it is not in turn actually divided into parts excited by different motions; and therefore in every body there are actually bodies infinite in number. (*A Specimen of Discoveries* [c.1686]; A VI 4, 1626 / LLC 323);

> I hold that matter is actually fragmented into parts smaller than any given, i.e. that there is no part that is not actually subdivided into

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32. This is important to stress since mereological addition is idempotent (A+A=A), the idea being that if I add A to itself I obtain nothing new. Therefore, the repetition of a measure must be interpreted as the sum not of the measure with itself, but of the measure with disjoint copies of itself (where a copy is an object congruent with the measure). On this point see MU 181; on Leibniz's theory of measurement, see Rescher (1955).

33. The importance of this procedure for the current argument and the need for idealization in its application have been emphasized also by Brown (2005: 463–469).

other parts exercising diverse motions (*to Des Bosses*, 11 March 1706; GP II 305 / LDB 33);

> But in real things, that is, bodies, the parts are not indefinite (as they are in space, a mental thing), but are actually assigned in a certain way, as nature actually institutes the divisions and subdivisions according to the varieties of motion, and … these divisions proceed to infinity … (*to De Volder*, 30 June 1704; GP II 268 / LDV 303).

We may describe the picture as follows: there are some motions in bodies that divide the body into actual parts. That is, different parts are individuated by different motions, since they are "excited by different motions." In this way, everything which belongs to the same part has a common motion.[34] Parts are homogeneous to the whole by definition, which means that they all have a finite magnitude.[35] These parts are contiguous, i.e. they touch each other but they do not share their respective boundaries. As a consequence, they are also disjoint. Moreover, they are co-integrant (which is Leibniz's term to indicate that all the parts together comprise the whole body). This implies that the division has produced finitely many parts $p_1, p_2,…,p_n$. If we now choose one of these parts—say p_1—then we must conclude that its magnitude is contained a finite amount of times in the (magnitude of the) whole body, otherwise the magnitude of the whole body would be infinite.[36] However, each of these parts are bodies in their own right, which means that they are divided into smaller parts by some further motions. Again, for the same reason, each of them has a finite magnitude, they are contiguous, disjoint, and co-integrant with one another: $q_1, q_2,…,q_m$ (wher $n < m$ since these parts, being contained in the former, are smaller than them). What we have obtained is a bigger, but still finite, number of smaller parts contained in the previous parts $p_1, p_2,…,p_n$. The same reasoning applies again and again: no matter how many times we reiterate the division, we shall always find parts with finite magnitude which are contained only a finite amount of times in the magnitude of the whole body.

That Leibniz had in mind such a picture of the division of bodies is clear from the following passage:

> So bodies are actually infinite, i.e. more bodies can be found than there are unities in any given number. The inference is obvious; for if we suppose any division to be made into only two parts, neglecting the others;

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34. On this topic, see Arthur (2014: 81) and Arthur (2018b: ch. 1).

35. No part can have an infinitesimal magnitude, in a non-Archimedean sense of infinitesimal.

36. This is because the whole body has a finite magnitude as well as the part p_1, and if we sum the magnitude of p_1 with itself infinitely many times, we obtain an infinite magnitude, not a finite one.

and we suppose only the second of these two parts to be subdivided, at least as many parts will be produced by this subdivision as there are divisions made: for example, if A=L+B, B=M+C, C=N+D, it is obvious that from the three divisions at least three different things are produced, L, M, N. (A VI 4, 1393 / LLC 237)

The moral of the story is that no matter how small a part is, since a part is homogeneous to the original body (indeed, it is a body as well), it will be contained a finite number of times in it, i.e. there is a ratio between the whole body and each of its parts. At the same time, however, there is no last division, which implies that there are actually infinitely many parts. Such a division thus presents a recursive structure: given a certain division of the body, there is always a further division with parts insides parts *ad infinitum*. At each level of division, the parts are disjoint (since they are contiguous); but when considering arbitrary parts obtained by different divisions, it is not generally true that they are disjoint. In fact, some of them will be sub-parts of others. When we consider all the actually infinitely many parts, these parts are not, generally speaking, disjoint. Since they are not disjoint, the fact that there are infinitely many parts does not bear on the magnitude of the body: we know that measurement consists in the repetition of disjoint parts (*partes extra partes*). This recursive division of bodies into finite parts is what allows a body to have a finite magnitude and, at the same time, infinitely many parts.[37]

Our explanation of the possibility for a body to have infinitely many parts and only finite magnitude hinges on the actual division of bodies which is *not* a division into infinitely many *disjoint* parts. However, one may object, this explanation cannot be right, because there is a case in which a division of a body into *infinitely many disjoint parts* is compatible with the body having a finite magnitude. This happens if the division resembles a convergent decreasing series, for example, the series 1/2+1/4+1/8+1/16+… which sums to 1. Indeed, Arthur (1999) uses exactly this example to argue in favor of the compatibility of infinitely many parts and finite magnitude.[38] Certainly, the analogy with infinite convergent series provided Leibniz with a useful heuristic to think of the division of bodies, since it shows that no contradiction follows from the attributes of "having infinitely many parts/terms" and "having finite quantity." But we have no reason to

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37. That the division of a body into parts has a recursive structure is also defended by Arthur (2018b: ch. 2).

38. "… I need to say few words on infinite series, since these serve Leibniz as a mathematical model for his idea of infinite division … A geometrically decreasing series, such as 1/2+1/4+1/8+1/16+…, has a sum, in this case, 1, as does any converging infinite series, such as Leibniz's own alternating series for π/4. In the same way, then, any part of a body can be further divided into infinity, and yet the body can have finite extent" (Arthur 1999: 109). Notice that Arthur's (2018b) analysis does not rely on the analogy with a converging infinite series contrary to what Arthur did in (1999).

think that this is anything more than a mere analogy, and that bodies are literally modeled after such series. One might observe that, given any infinite division of a body into parts (which are not generally disjoint), it is always possible to select an infinite subset of them such that they are *disjoint*, co-integrate, and modeled as an infinite convergent decreasing series. But this requires a selection between the actually existing parts, and so this fact cannot be used to argue for the idea that the division of bodies is literally modeled after these series. On the contrary, the explanation based on the recursive structure of bodies that I have developed here provides us with *a general reason* that explains why the two attributes are compatible in the case of all bodies.

This recursive structure allows a body to have a finite magnitude while at the same time possessing infinitely many parts. It therefore allows the application of the measurement procedure to bodies. The Part-Whole axiom is embedded into the measurement procedure, because if the subtraction of a part did not affect the whole (giving us a remainder strictly smaller than the whole), then we could go on subtracting parts forever without anything happening, and the procedure would not yield any result. This unequivocally shows that bodies respect the Part-Whole axiom. Since each part is finite, if we subtract it from the entire body, the remainder will be smaller than the original body. Therefore, there are no reasons to dismiss the identification of bodies with wholes (with parts).

4.4 No Measurement of the Universe

Contrary to bodies, the universe does not have a recursive structure that would allow it to have a magnitude. Indeed, the universe is the aggregate of all substances, which are infinitely many. But substances are like worlds apart, i.e. if we think of the universe as a whole whose parts are the substances, then they would correspond to a division of the whole into infinitely many disjoint parts. For this reason, the measurement procedure cannot be applied to the universe, and on the supposition that it formed a whole with parts, it would violate the Part-Whole axiom. The universe is no whole, and consequently has no magnitude.

If we consider the universe not as the aggregate of all substances, but as the aggregate of all bodies, the result does not change. Indeed, Leibniz is explicit in the claim that we cannot conceive the world as a single body:

> Thus, we may indeed call all bodies together "the world," but in reality the world is not some one thing, but this alone can be said: for any given body, there is some larger one in the world and we never reach a finite body that includes all [bodies]. Nor, however, is there such an infinite body. (A VI 4, 1469 - circa 1683–85).

Even in the case in which we conceive of the world as the aggregate of all bodies, still it does not have the same structure of a body.

This seems to get to the bottom of the difference between an organic body and the universe. The latter violates the Part-Whole Principle, while the former is compatible with it. If there were a universe, it would be infinite both in multitude and in magnitude. By contrast, the body is infinite in multitude but not in magnitude. However, infinity in multitude can be dealt with syncategorematically, i.e. Leibniz has a precise way of squaring his rejection of infinite number with infinity in multitude. And this is what allows him to accept bodies as wholes, but to reject that the universe is a whole. This rejection amounts to the claim that the universe is really a plurality of things, not one single entity.

What the argument against the soul of the world suggests is that respecting the Part-Whole Principle is a necessary condition for something to be the body of a soul. Why this is the case is the focus of the next section.

5. On Premise 1

Premise 1 is the following: *if the world has a soul, then it is a whole.* In this section, I am going to explain this seemingly awkward premise. I argue that this premise follows from the idea that if the world has a soul, it is a body of a living organism, and since it is a body, it is a whole (with parts). I shall argue that this argument is rooted in Leibniz's conception of the world soul as something immanent to things. If the world soul were to exist, it would be immanent to things, and thus it would not be a purely active being, but it would also contain a material passive element (expressed by its body).

To explain premise 1, it is useful to start with a passage from a letter to Des Bosses of 11 March 1706:

> The actual infinite in magnitude cannot be exposed to view in the same manner as (the infinite) in multitude. … Hence even if the world were infinite in magnitude, it would not be one whole, nor could God be conceived, with certain of the ancients, as the soul of the world, not only because he is the cause of the world, but also because such a world would not be one body, nor could it be regarded as an animal, nor, indeed would it have any but a verbal unity. It is therefore an abbreviated way of speaking when we speak of one thing where there are more than can be understood in a single assigned whole, and we treat like a magnitude that which does not have the properties of a magnitude. (GP II 304–305; trans. from Brown 2005: 450)

This is an extremely rich passage. It surely confirms our analysis of bodies according to which what matters is their finite magnitude, and not how many parts they have. However, more importantly for premise 1 is Leibniz's claim that what is not *one whole* is not *one body*, nor is it an animal. The idea – that we have also seen in the passage from the *Theodicy* quoted in §3 above—is that denying that the world is a whole implies denying that it is an animal, i.e. that it is the organic body of a living organism endowed with a soul. The same idea is present in A VI 4, 1492: "… but if it is assumed that the world is infinite, it is not one being, i.e. a body one in itself (*unum per se corpus*)." This suggests that the notion of body plays a key role in Leibniz's understanding of premise 1. Indeed, the argument that emerges from these passages is the following:

> **1a.** If the world has a soul, then it is a body.
> **1b.** If something is a body, then it is a whole.
> **Conclusion (Premise 1):** If the world has a soul, then it is a whole.

The notion of body works as the middle term that links the two premises into the conclusion. Indeed, in this reconstruction, the notion of body is what explains the connection between "having a soul" and "being a whole." Notice that premise 1b is the main claim that we have defended in §4. If one does not adhere to our previous analysis (maybe because one thinks that bodies cannot be whole since they have infinitely many parts), then one cannot avail oneself of this justification of premise 1, and consequently, one cannot appreciate its deep meaning. Without 1b there is no link between 1a and the conclusion (premise 1).

The passage just quoted does not provide us with a justification of 1a. However, its justification emerges from what Leibniz tells Clarke: "If it is by means of a sensorium that God perceives what passes in the world, it seems that things *act on him* and that therefore he is what we mean by a soul of the world" (LC 55, emphasis added). 'Soul of the world' denotes here something which is not only active, but also passive, since things can act on it. Passivity is here treated as a constitutive element of the soul of the world. We already know from §2 above that Leibniz conceived of the world soul as "a universal nature immanent to the things" (GP II 383), and not as something that transcends the world. It is thus natural to speculate that this passive element is just a consequence of the immanent nature of the world soul. This finds confirmation in what Leibniz adds to Clarke a few lines later: "Will not this doctrine, moreover, tend to make God the soul of the world, if all his operations are natural like those of our souls on our bodies? And so God will be a part of nature" (LC 62). If God were the soul of the world, he would be a part of nature, and his operations would be like those of our souls in relation to our bodies. Implicit here is the view that the world would be the body of such a soul, and that the model for the relationship between the

world and its (alleged) soul is given by the relationship between our body and its soul. The view is that the immanent nature of the world soul makes it partake in the passivity of the material world. Indeed, passivity is often accounted for by the presence of a material ingredient in the created substances, i.e., by the fact that each created substance is united with a body. It is the immanent nature of the world soul that requires that, if it existed, it is united with a body.

That the passivity requires the presence of a material body is something explicitly stressed by Leibniz in other contexts. In the preface to the *New Essays*, Leibniz affirms that

> I agree with most of the ancients that every Spirit, every soul, every created simple substance is always united with a body and that no soul is entirely without one. (NE 58)

He then explains why he believes that each created substance is united with a body in Book II, when Philaletes (Locke's spokesman) says that "hence may be conjectured, that created spirits are not totally separated from matter, because they are both active and passive. Pure spirit, viz. God, is only active; pure matter is only passive; those beings that are both active and passive we may judge to partake of both," and Theophilus/Leibniz replies that "these thoughts are greatly to my liking, and perfectly express my own view, just so long as the word 'spirit' is constructed broadly enough to cover all souls or rather … all entelechies or substantial unities … " (NE 225).[39]

Created substances are both active and passive. The principle of passivity is matter, which means that every created substance, insofar as it is passive, must be united with a material body. Since God is purely active, God has no body. Therefore, passivity is one of the features that distinguishes the creatures from the creator: the same is thus true for the property of having a body (if the latter should account for the passive elements in substances).

The relationship between the substances and their own body is a very complicated issue in Leibniz's thought; addressing it would require (at least) a paper on its own. Fortunately, for our present aims, we do not need to deal with this issue; rather, it is enough to point out that premise 1 presupposes that, if it existed, the world soul would have an organic body, because, by definition, it would be something immanent to the world, and so it would partake in the passivity of the material world. Leibniz's understanding of the world soul concept implies that the supposition that the world has a soul amounts to the supposition that the world is the body of this soul.

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39. A similar passage can be founded in the correspondence with Clarke, where Leibniz remarks that "I hold that every created substance is attained with matter" (LC 16).

One may object that if the soul of the world has a passive element, then it must be a created substance, and consequently, the argument already presupposes that God is not the world soul. The argument would merely be an argument against the idea that there is a created world soul, not against the idea that God is such a soul. But this seems plainly wrong, since when Leibniz presents the argument, he claims that "God cannot be understood as the world soul: not the soul of a finite world because God himself is infinite, and not of an infinite world because an infinite body cannot be understood as one entity" (A VI 4, 1509). If the argument only excluded a created world soul, then Leibniz could not conclude that there is no soul of the world *tout court*, because he could not exclude that God is the soul of the world. But this objection is misplaced. The strength of the argument is that it does not presuppose a specific conception of God. Indeed, Leibniz is arguing against his opponents that, whatever their conception of God, if they identify God with the soul of the world, then God would be immanent to things and so would be partially passive and will consequently have a body. In other words, the passivity would be a direct consequence of the hypothesis that God is the soul of the world. But then Leibniz can exploit the argument to show that the world is not a whole, and so it cannot be a body of a soul. So, there is no soul of the world, and *a fortiori* God cannot be identified with it. It is true that Leibniz can appeal to other arguments, such as the fact that God is purely active and thus cannot have any element of passivity, and for this reason he cannot be the soul of the world; but such an appeal is based on a certain conception of God, and if his opponents do not share it, then they are not forced to accept Leibniz's conclusion.

6. On Unity, Quantity, and Body

We have argued that everything that is the body of a substance is a whole. This must be the case precisely because otherwise a contradiction would be realized. Suppose there were a soul of the world. Then the world would be an organism with an infinite body (in magnitude). God would have created a substance with a body which would not respect the Part-Whole Principle. This is a contradiction. This contradiction is given by the infinite magnitude of the world, which would be an instantiation of a categorematic infinite. The fact that the world has infinitely many constituents is not enough to produce a contradiction, because Leibniz can interpret it in a distributive way, by saying that, given any (finite) number of terms y, the world has more constituents than y. But no correspondent solution is available with regard to infinite magnitude. Leibniz must therefore deny that anything with an infinite magnitude is possible. If the world were an organism endowed with a soul, it would have an infinite magnitude; hence the world is not an organism.

Here we can see a subtle connection between the concepts of *whole* and *one/ unity*. Already in 1672, when he first rejected infinite number, Leibniz had concluded his argument by stating "that infinity itself is nothing, i.e. that it is not *one* and not a *whole*" (A VI 3, 168 / LLC 9; emphasis added). In this passage Leibniz explicitly states that the rejection of infinite number means that it is not a whole, but *also* not a unity. Clearly, the notion of unity involved here is not substantial unity, as should be clear form what we said above. Moreover, also finite wholes, having parts (and so being complex and divisible), are not substantial unities. To deny that the infinite is one means simply to deny that it is something, i.e. that there is an infinite object (a categorematic infinite). While each body is one, the world is not one, but rather many, i.e., it is a plurality of substances.

In this sense, premise 1 expresses a necessary condition for something to be the body of a soul. To be a whole implies being one (object). What really matters is not how many parts something has, but that the object in question can be amenable to measurement via the generalization of the Euclidean algorithm. If this is the case, then the Part-Whole Principle is respected. At this point, two observations are in order.

First, note that the same rejection of an infinite number is due to the impossibility of an infinite magnitude, not an infinite multitude. In fact, infinite multitudes are legitimate as far as they are interpreted distributively. However, the alleged infinite number is viewed in line with finite (positive) integers: it is an aggregate or whole of unities, such as each integer is. For there to be a number corresponding to an infinite multitude, this multitude must be treated as an infinite whole with an infinite magnitude, with a consequent violation of the Part-Whole Principle. Therefore, it is the rejection of infinite magnitude (based on the Part-Whole Principle) that triggers the rejection of infinite number.

This centrality of the notion of magnitude is confirmed in the mature Leibniz by the role played by the measurement procedure described above. Indeed, the Part-Whole Principle is a necessary condition for the procedure (applied to line-segments or other geometrical figures) to get off the ground. The procedure determines magnitudes, not cardinalities of multitudes. However, Leibniz is clear that we can justify (positive) real numbers in terms of this procedure.[40] Leibniz's mature conception of number explicitly relies on the procedure to measure magnitudes; indeed, numbers express the ratios between magnitudes. We

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40. See *Initia rerum mathematicarum metaphysica* where, after expounding the measurement procedure and developing the sequence of quotients obtained by applying the procedure in the form of continuous fractions, Leibniz writes that "from this it is clear that we can define the general notion of number, integer, fractus, rational, surdus (irrational), ordinal, transcendent, as what is homogeneous to the unity, or what is in relation to the unity like a straight line to another straight line" (GM VII 24).

have seen that this procedure consists in determining how many times a measure is contained in the measured thing. This is completely analogous to the fact that a positive integer consists in how many abstract units it contains. Since, for Leibniz, an integer is a collection or whole of unities, we may say that an integer is "measured" by the units it contains.[41] We may say that it is the mereological framework in which Leibniz's argument is developed that pushes him to prioritize the notion of quantity as magnitude and the idea of measure over the notion of quantity as multitude and the idea of counting pluralities of objects (counting in the sense of the cardinality of a multitude).

Second, quantity is not something that only pertains to *our conception* of the world; it is something that directly pertains to the world. Indeed, it is the violation of the Part-Whole Principle, one of the key defining axioms of quantity, that allows Leibniz to deny the existence of a soul of the world. When dealing with the created world, quantity must be considered a fundamental aspect of it.

The world has no soul and so it is not an organism because to suppose otherwise would violate the Part-Whole Principle. This principle is exploited as a metaphysical constraint that determines what can and cannot be a body, and so what can be endowed with a soul. Since the Part-Whole Principle is one of the key conditions that something must respect to be a quantity, this implies that quantity turns out to be a central feature for something to be a body of a substance. We thus have here an argument for the claim that all bodies have quantity:

A. Everything which is a body of a substance is a whole.
B. Any whole has quantity (Part-Quantity).
Conclusion: Everything which is a body of a substance has quantity.

This interpretative line gives a key role (in the created universe) to the notion of quantity. Since everything created is a substance and every substance has its organic body (which is an aggregate of further substances), it follows that, insofar as it has a body, each created thing has quantity. The measurement of quantity is a comparative process, and thus it allows us to determine the quantity of one object only *in relation to* some other object (the measure); but this does not make the quantity of an object purely relational, as if an object has a quantity only when it is compared with something else.

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41. Since counting a multitude of objects is understood by Leibniz under the light of his theory of measurement (which is primarily a theory for measuring magnitudes)—in those cases where bijection (i.e. the principle of cardinality) contradicts the results of the theory of measurement (which happens in the infinite cases, but not in the finite ones)—Leibniz is forced to reject bijection as a standard for equality, and so he is forced to reject infinite numbers.

7. Comparison with Existing Interpretations

It may be useful at this point to step back and compare the present proposal with others'. Harmer (2014: 245) rejects both Carlin's and Arthur's solution because "the mistake that both make is that they attempt to distinguish the world-body from an organic body in terms of unities. But there is a more important reason to resist these solutions: they inaccurately construe Leibniz's rejection of infinite number as a result primarily about unity." What Harmer argues is that the argument is about wholes, not unities. Harmer then rightly distinguishes genuine wholes (those composed of finitely many parts) from fictional wholes—we may say "mere aggregates"—(those composed of infinitely many parts). However, Harmer believes that this distinction makes it unnecessary to provide an explanation for the difference between the world and an organic body: "Given the preceding analysis of wholes … . I do not need to develop a relevant difference between the world-body and the body of a substance. It follows from what I have established above that composite substances, insofar as they are substances, will not be wholes at all. … In virtue of having infinitely many parts, the body of a composite substance will be merely a multitude, not *one whole*" (Harmer 2014: 254). But this cannot be right: first, this interpretation obliterates the fact that Leibniz is very clear in stating that the world has no soul because it is not a whole; second, as argued above, bodies can be said to be wholes despite having infinitely many parts.

Harmer, however, is right in claiming that the argument is primarily about the notion of whole, and not unity, contrary to what Carlin and Arthur had held. However, to dismiss these proposals for this reason seems to me too hasty. Carlin's and Arthur's solution centered around the notion of arithmetical unity. Their idea is that a body, despite not being a substantial unity, is an arithmetical unity, while the world is not (I take the notion of arithmetical unity to denote the fact that the magnitude of a body is finite). Arthur develops this point further by suggesting the analogy between the division of a body with a convergent decreasing infinite series. While the potential but also the limit of this analogy has been dealt with before, it seems to me that both Carlin and Arthur were on the right track by pointing out the role that magnitude plays in the argument. However, here is the point at which my analysis goes further, insofar as it points to the reasons (the arrangement of parts in a body) why bodies have a finite quantity, while the world has none. In addition, Harmer (2014: 244), always discussing Carlin and Arthur, had complained that "there is no basis for concluding either that 'arithmetical unity' tracks a certain degree of accidental unity or that a difference in degree of accidental unity can account for why some collection—i.e. the organic body—can have souls while others—

i.e. the world-body—cannot." Harmer's claim here is based on the passage that Carlin and Arthur had relied on for proposing the idea of arithmetical unity as a solution to the problem.[42] I believe that Harmer has a point in stressing that the passage where Leibniz appeals to arithmetical unity does not give us any clue as to why this feature matters. This clearly shows the limit of appealing to the notion of arithmetical unity. But my proposal does not appeal to it, and it is therefore immune from this criticism. Moreover, there is a clear sense in which "degree of unities" can explain why some collections have souls while others do not: having a finite magnitude (therefore being a whole) is a necessary condition that each body (of a substance) must respect, otherwise we would obtain a contradiction.

Finally, Brown (2005: 469) has the merit of being the first (and until now the only one) to explicitly appeal to the measurement procedure for quantities, stressing that the problem regards being infinite in magnitude, not in multitude: "for given that Leibniz is denying that an infinite world is not a *whole in magnitude*, he can be interpreted as saying that the magnitude of the space that such a world would have to occupy would be infinite – that is the number of congruent parts into which it would have to be divided in order to determine its magnitude would be infinite." However, he uses this argument to make a distinction between the world – *unum per accidens* – and an organic body – *unum per se*: "But since no infinite can be considered a whole without that implying a contradiction, an infinite whole cannot be a *whole in magnitude*, which implies that it cannot be a *unum per se*" (Brown 2005: 469). Here—in interpreting the passage to Des Bosses where Leibniz claims that since the world is infinite, "it is not one being, i.e. a body one in itself (*unum per se corpus*)"—Brown identifies bodies with *unum per se*. The problem is that with '*unum per se*' Leibniz usually denotes substantial unities. Here I believe that Brown is conflating the unity proper of a whole with substantial unity: contrary to the standard use of the locution '*unum per se*', in this passage '*unum per se corpus*' simply denotes the fact that a body can be treated as a single object since it can be treated as a whole (with parts). From a metaphysical point of view, mere bodies (i.e. bodies independently of their dominant monads) are aggregates, pluralities, not unities. Being a whole is not being a substantial unity. Indeed, since substantial unities are simple, they have not parts and so they are not wholes. But mere bodies can be treated as wholes, and so as one entity: mere bodies are entities, in the sense that they can be bodies of something (i.e. a dominant monad) that has enduring, and not merely accidental, unity.

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42. This is the passage: "A fraction of an animal, or half-animal, therefore, is not a being per se, since this can be understood only of the body of the animal, which is not one being per se but an aggregate, and has an arithmetical unity and not a metaphysical unity" (LDB 31).

8. Conclusion

In this paper, my aim was to provide a solution to the problem raised by Leibniz's mathematical argument against the soul of the world. On the one hand, the world cannot have a soul because it is infinite; on the other, each body is infinite but can have a soul. Can Leibniz consistently reject the existence of the world soul? The reconstruction and the analysis of the argument that I have offered shows that he can. It is not important how many parts a body has, since, given that they are arranged recursively, they sum up to a finite magnitude. Bodies can be treated as wholes with parts. By contrast, the world has no magnitude, since the arrangement of its alleged parts would require this magnitude to be infinite, contradicting the Part-Whole Principle. The argument thus is centered on the notion of magnitude. It is infinite magnitude that Leibniz is rejecting when rejecting infinite wholes.

The fact that bodies can be treated as wholes with parts is crucial to fully understand premise 1: why must something endowed with a soul be a whole? The answer is simply that something with a soul is a body (of a living organism), and each body is a whole with parts. For this reason, to deny that something is a whole implies denying that it is the body of a soul. If we deny that the world is a whole, it cannot be the body of a soul, and given that the world soul is immanent to things, we can conclude that there is no soul of the world. Premise 1 thus relies on Leibniz's own conception of the nature of the world soul and on the idea that bodies are wholes.

Being a whole is thus a necessary condition for something to be a body of a substance. If this were not the case, we would admit the possibility of bodies which are not finite wholes, i.e. bodies infinite in magnitude. But a body infinite in magnitude would be an instance of a categorematic infinite, and so it would contradict the Part-Whole Principle.

Acknowledgements

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