UMURJ
University of Michigan Undergraduate Research Journal

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Dear Readers,

Our mission at UMURJ is to give students the opportunity to present and publish their research. You will find that, compared to previous years, this issue contains less entries and illustrates the impact that the Covid-19 pandemic has had on our campus. Although the pandemic has posed challenges for student researchers, our publication received great manuscripts presenting original research done by undergraduates. This issue contains several social science and life science papers entries that have been reviewed by faculty members here at the University of Michigan. Our publication also features a student interview done by one of our editors that offers insight into an undergraduate's research journey.

Our publication requires collaboration among authors, editors, faculty, and staff. This collaboration ensures that we can provide a platform for a variety of academic fields and support interdisciplinary explorations. Our staff works closely with the Undergraduate Research Opportunity Program, the Michigan Library, and the Michigan Office of Research.

The three of us have had the privilege of editing for the journal for the past three years. Each of us are passionate about our respective fields and excited to share this platform with an amazing team. The past year has been challenging for members of the university community, and another challenging year awaits us as the Covid-19 pandemic continues. Despite the circumstances, we are committed to building a stronger research community at the University of Michigan and will continue moving forward in full force.

Sincerely,
Aniket Dagar, Ayse Eldes, Madhulika Shastry
UMURJ Co-Editors-in-Chief (2021–22)

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Our Sponsors

UROP

The Undergraduate Research Opportunity Program (UROP) creates research partnerships between undergraduate students and University of Michigan researchers and community partners. All schools and colleges are active participants in UROP, which provides a wealth of interesting research topics for program participants. UROP started with 14 student/faculty partnerships in 1988 and has expanded to include more than 1300 students and 800 faculty researchers.

The purpose of the Undergraduate Research Opportunity Program (UROP) is to support the mission of the University of Michigan (U-M) by engaging undergraduate students in research, scholarship, and creative inquiry. UROP encourages students towards a life-long appreciation for discovery and social benefits of diversity, for building understanding across differences, and for critically examining information in the world around them. By providing opportunities for students to work on research and creative projects with University of Michigan researchers and community partners in all academic disciplines, UROP creates conditions for an equitable, inclusive, and supportive educational environment where every person feels valued and has an opportunity to add significance.

UMOR

With 19 schools and colleges and more than 200 centers and institutes, the University of Michigan is leading in research in virtually every major area of science, engineering, medicine, business, arts and humanities, public policy, and education.

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The U-M Office of Research catalyzes, supports and safeguards research across all three campuses so that students, faculty, and staff can advance knowledge, solve challenging problems, create new scholarly works, and enhance quality of life.

As the nation's largest public research university, the U-M Office of Research works with leaders across campus to cultivate interdisciplinary research, diversify sources of funding, improve the efficiency of research operations and strengthen partnerships with industry, government, and peer institutions worldwide.

UM Library

Consistently ranked as one of the top ten academic research libraries, the University of Michigan Library makes available an extraordinary array of resources and services. Our mission is to support, enhance, and collaborate in the instructional, research, and service activities of the faculty, students, and staff, as well as contribute to the common good by collecting, organizing, preserving, communicating, publishing, and sharing the record of human knowledge. Expert staff are committed to helping learners and scholars tap into the full potential of these informational resources and to providing a full spectrum of assistance for research and teaching. We collaborate with students at every step in their educational career and engage with faculty and graduate students research. We provide a hub for scholarly publishing via Michigan Publishing that is responsive to scholars. Connecting students, the public, and multiple disciplines through publication of non-technical articles is opportunity to showcase student excellence and public engagement. We are delighted to share our expertise and resources with campus partners and students as part of the University of Michigan Undergraduate Research Journal.

Thanks to our Faculty Reviewers

We thank the following faculty for their contributions to ensuring quality work in the undergraduate research journal and providing guidance through feedback for all of our journal submissions.

Social Science

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- Dominick Bartelme, PhD

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We are a student-run, non-technical research journal. Our mission is to build connections between undergraduates, graduate students, and the public, as well as among the different academic disciplines through the publication of non-technical articles in all fields of research. Our goal is to tap into the sea of student creativity and effort in all academic disciplines through a peer-edited, faculty-reviewed electronic and print publication. We hope to inspire interest in research not only in our student body here at the University of Michigan, but also to undergraduates and faculty all around the nation.

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I recently had the opportunity to sit down with Avery Heo, a third-year student at the University of Michigan. She is studying neuroscience and plans to attend medical school after taking a gap year or two.

As a sophomore Avery participated in the Undergraduate Research Opportunity Program, or UROP. This program is offered through the College of Literature, Science, and the Arts (LSA), and students from colleges all around the nation are welcome to apply, helping create far-reaching partnerships between undergraduate students and expert faculty members. She tells me that her favorite part of this program was how it was structured. Each student has a peer facilitator to help mentor them through the process, and the students submit reflection pieces to check in and make sure that they are enjoying the research experience. She says she also really enjoyed the different workshops she was able to attend, especially the resume-building workshop. It was through UROP that Avery found a research lab at the Kresge Hearing Research Institute, where she works to establish a behavioral test for hidden hearing loss in guinea pigs.

When I asked why she chose to participate in undergraduate research, Avery's response was twofold. "As classes become more advanced and more specialized," she tells me, "it is really rewarding to be able to use the information that I am learning about in class and apply it to real projects." Research labs, such as those through UROP, give many students the opportunity to do just as Avery describes: take class concepts and use them in real-life scenarios. She also tells me that working in a research lab gives her great experience and preparation for her future in medical school.

I was very curious to understand how working in a research lab during the COVID-19 pandemic differs from how it was working there the year before. Avery tells me that while many aspects are different, several features that are the same. Instead of attending her weekly journal club and lab meetings in person, she signs onto Zoom and participates virtually. "A lot of the data analysis that we do can also be done remotely which is really nice," she tells me. While her meetings have all been moved to a virtual format, Avery is still able to go into her lab twice a week and work in person. "This year in particular," she says, "it is especially nice to have a set routine of days that I go to the lab, and I really like having something in-person when all my other classes are completely online."

Contact: Sarah Tolchin

I know being a research assistant while taking a full load of academic credits can be very time-consuming and overwhelming. Because she is planning on attending medical school after she graduates from U of M, Avery is also balancing studying for the MCAT on top of her research and classwork. So, I asked her what her tips and tricks are to staying on top of her workload. "Backpacking days before your registration date is the key," she tells me. "I always make sure to leave two to three days either completely open or with fewer hours in class to make sure that I can go into the lab at least four hours on those days." This past semester, Avery left her Mondays and Fridays class-free and went to her lab from nine in the morning to one in the afternoon.

At the end of our conversation, I asked Avery if she had any advice for both incoming and current students who want to get involved with undergraduate research. "Don't be afraid to cold email," she says. "Reaching out to faculty is so important because no one will know that you are looking for an opportunity in a lab unless you contact them." Moreover, it is always possible that labs are looking for undergraduates and might just not be publicly posting about the opening. She also says that when you cold email a research faculty member, make sure to have an updated resume and good explanation as to why you are interested in research and in that project in particular. "If you are an incoming student or a current freshman," Avery says, "you should definitely look into participating in UROP because that's what helped me make my own connections to research opportunities on campus."

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Analyzing the Behavioral Motivations behind Israel's Water Conservation Efforts

KENWOO KIM AND CHARLES MORPHIS

Israel has been at the forefront of water conservation since the 1950's. Effective water technology, management, recycling, and production allows the nation to provide water for its 9 million inhabitants, despite the limited water resources and climate. This research offers a specific look into Israel's behavioral use of water and cultural position regarding the resource while also examining multiple unique areas and their relationships. This research utilized interviews of water experts and a survey from respondents largely composed of those water experts. The results revealed Israel's religion, education, media, and interpersonal interactions all contributed to a water conscious culture that is further enforced by government centric water laws, water pricing, and technological support. These findings could provide a comprehensive guide to be utilized by future policymakers to incentivize and develop water conscious cultures within their own communities.

Literature Review

Siegal (2017) is the definitive, foundational text for this research, offering a broad but detailed view into Israeli efforts in water conservation. However, the text did not specifically address the motivating behaviors behind water conservation, and this research aims to fill this gap. Lipchin (2007), Dinar (1998), and Minahem (1998) discuss Israeli water culture, pricing, and policy respectively and are representative samples of area specific research into Israeli water usage. Yet, these texts and texts like these often omit related, broader ideas that would help further define and contextualize Israeli water behavior. For example, Lipchin (2007)

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provides an excellent analysis of the relationship between Israeli water culture regarding Zionism and agriculture but does not address urban water culture.

Methodology

In developing this research, 25 interviews were conducted throughout Israel from June 11th to July 8th, 2019. The majority of these interviews were conducted in English and in person. However, two interviews were conducted via phone and two other interviews were conducted in Arabic, facilitated via an experienced translator. These interviews began with a short list of fixed questions and then shifted to more pointed and specialized follow-up questions, depending on the interviewee's subject specialty. Many of the interviewees provided an expert opinion on water conservation and water policy. The majority of participants worked in academia, while others had experience in private sector water-professions, municipal government, and agriculture. A representative sample of a few key individuals include: Dr. Gideon Oron, Professor Emeritus in Ben-Gurion University of the Negev's Department of Hydrology and Microbiology; Mr. Oded Revivi, Mayor of Efrat; Mr. Hagai Ido, Chief of the Economics Department at Mekorot; and Mr. Amir "Chaki" Peleg, a cherry tomato farmer in the Western Negev.

In addition to the interviews, this research compiled a 44-question survey focused on water experts, largely composed of our interviewees, in order to consolidate several key data points and represent the opinions of an expert populace. Surveys were distributed via a digital medium and translated into Hebrew and English. From this point forward, this survey will be referred to as the expert survey.

Results

The interviews discussed a variety of topics and provided the foundation for much of this research. The main themes of these interviews can be organized into 10 areas: religion, education, media, interpersonal interactions, laws, water pricing, climate, water scarcity, history, and technology. Water pricing was mentioned the most often with 56% of interviewees discussing the subject. Technology was discussed in 44% of interviews and media followed with 36%. Both education, interpersonal interactions, and water scarcity were discussed with the same frequency at 32%. History was brought up in 25% of interviews and laws in 20%. Religion and climate were discussed the least and were only present in 16% of interviews.

From the expert survey, which garnered 26 responses, 52% of self-identifying respondents work in Education and Research, 14% work in private sector professions related to water policy or technology, 5% are farmers, and 29% come from other professions such as municipal officials, retirees, and blue-collar workers. The gender ratio is 58% male and 42% female, with the majority of respondents possessing a graduate degree or higher. Roughly 19% possess a high school diploma. The majority of respondents are from the age range of 41–60, with outliers on both sides, and the next largest group being 31–40. About half of respondents come from households of 4–5, with 23% coming from 6–7 and 23% coming from 2–3. Only 1 respondent lives alone. The supermajority of respondents come from rural, Southern communities. Most live in the Arava, Negev, or Sinai. However, Jerusalem, Eilat, and the West Bank are represented. The supermajority of respondents are non or slightly religious Jews.

Discussion

History

Between 1929 to 1939, Britain's Mandatory Palestine experienced a large wave of Jewish immigration, bringing the population to nearly half a million people and triggering the 1936–1939 Arab Revolt. In response, British Parliament issued the White Paper of 1939 which limited Jewish immigration and land expansion in the Negev Desert. Britain cited limited water resources to justify the restriction. To refute this claim, water engineer Simcha Blass developed a comprehensive water plan for self sufficiency, outlining a blueprint to drill for aquifers in the Negev, divert rivers, and create a national water carrier (Siegel, 2017, pp. 20–27). Zionist desires to increase immigration depended on the success of the water plan. In 1946, Zionist leadership showcased the plan's feasibility by establishing permanent farms with deep drilling capabilities in the Negev and discovered aquifers. Blass further developed the Champagne Pipeline, a regional water transport system. In 1948, these actions allowed the newly formed nation of Israel to claim the sparsely inhabited Negev and open up land and resources for more immigration (Siegel, 2017, pp. 30–33). From this operation, the Zionist led survival and expansion of Israel became inextricably tied to water availability.

By the 1950s, Israel began making headway in novel water technology and practices. When no other countries considered utilizing wastewater, Israel began construction of the Shafdan sewage treatment facility in 1956, with hopes of recycling the water for future agricultural usage. Now Israel recycles over 85% of wastewater and is the leader in wastewater recycling (Siegel, 2017, pp. 78–81).

In 1959, Simcha Blass developed micro- or drip irrigation. This novel irrigation technology utilized 50–60% less water than conventional flood or sprinkler irrigation. Furthermore, it provided higher yields (Siegel, 2017, pp. 58–59). The nascent country embraced and widely utilized drip irrigation, setting a historical precedent for creating groundbreaking water technologies. Israel now annually exports $2.2 billion in water technology and consultation, with a predicted potential of $10 billion in a few years (Siegel, 2017, p. 170).

In the 1970's and 1980's, Israel faced a renewed water shortage and began reducing the consumption of water. However, water management could only mitigate the demand but not produce new sources of water. For a decade, debates to solve the issue were held, even considering importing water from Turkey, and the nation ultimately opted for large scale desalination despite high initial costs (R. Kasher, personal communication, June 18, 2019). In 1997, the first desalination plant was built in Eilat and was followed by several other plants, each larger and more sophisticated than the last. Desalination now provides the majority of municipal water in Israel and generates upwards of 600 million cubic meters of potable water per year (J. Sack, personal communication, July 3, 2019). Israel opted to pay the heavy upfront costs of desalination plants in order to provide long term, sustainable water sources. The young nation is the only country in the world to reverse desertification (O. Revivi, personal communication, June 23, 2019).

Geography

The dry climate and limited natural sources of water in Israel have deeply instilled the concept of water scarcity into Israeli society. Israel can be divided into two climates: the Mediterranean climate in the north and coastal west and the arid climate of the south. In the south, rainfall averages 1 inch annually. The cities of Sde Boker, Beersheba, Arad, and Kiryat Gat represent the northern portion of the arid south, with higher annual rainfall, but only receive 3–10 inches of rain per year. A minimum of 20 inches of water or 6000 mm per hectare per year is needed to grow crops in this region, requiring farmers to draw water from other sources (G. Oron, personal communication, June 11, 2019). Southern Israel is a desert, with scarce precipitation that makes rain a notable event of celebration and rarity (J. Zvuloni, personal communication, June 17, 2019). In the desert town of Merhav Am, the residents collectively remember the last major rainstorm that occurred over a decade ago. All activity ceased, with schools releasing students early, and children and adults alike collectively embracing the puddles, streams, and rainfall formed by the passing storm (O. Ben Zeev, personal communication, June 30, 2019).

The Sea of Galilee, or locally known as the Kinneret, is one of the few large sources of surface freshwater in the nation. As one of the few visible sources of water, the Kinneret once provided 20% to 25% of the nation's drinking water and is a symbol of survival, even maintaining religious importance to this day (O. Nir, personal communication, June 26, 2019). The Coastal Aquifer, Mountain Aquifer, and Negev and Arava Aquifer are the primary sources of groundwater, but the aquifers, Kinneret, and precipitation alone cannot sustain Israel's current population. Life in Israel is only possible through water conservation practices, recycled wastewater, and desalination. The Mediterranean climate of the north and coastal west experience rainless summers but short rainy winters. The majority of rainfall in Israel occurs between November and March with the majority falling in this northern region. In short, Israel has limited water resources, requiring strong measures to encourage conservative use.

Culture

Cultural influences play an enormous role in Israelis' day-to-day water consciousness and conservation practices. They indirectly and directly alter Israeli behavior to use less water and to further seek new avenues to save water. Cultural effects can be broken down into the areas of religion, education, media, and interpersonal interactions.

Much like its history, Israel's water conscious culture stems from its religious teachings. A telling quote from Seth Siegal's *Let There Be Water* draws a connection of Judaism and water, " 'Rain' is not only mentioned nearly one hundred times in the Jewish holy book, but there are even specific Hebrew words – still in use in modern Hebrew – for the first and last rainfalls of the year. If Eskimos have multiple words for snow because of its constant presence, Jews in the Holy Land would seem to have several words for rain because of its scarcity." In Judaism, prayers for rain can be invoked in times of drought and hold importance on religious holidays, such as Shemini Atzeret in which a prayer marks the inception of the rainy season. Rain's importance in Israeli society even produces specific and unique prayers to the nation. One such prayer translates to "don't look into the prayers of the ones who are walking," and essentially asks to ignore prayers of people outside who want to avoid rain (E. Av, personal communication, July 2, 2019). Beyond rain, water has been tied to important religious figures. Influential 1st century Jewish scholar Rabbi Akiva was inspired to study the Torah after observing water droplets erode a hole into a rock and concluding if water had the power to steadily dissolve rock, he could have the diligence to study Judaism at the age of 40. In the Jewish Bible, it is believed that when the Messiah arrives, Israel will have plenty of water (O. Ben Zeev, personal communication, June 30,

2019). Rain prayers and religious figures' association to water are only two of the many facets of how Judaism values water. This religion provides a basis of respecting water as a resource that can be further developed through education.

In Israel's education system, children are socialized to respect and embrace water conservation, especially at an early age. From the expert survey, 61% of respondents received a curriculum on water conservation for at least 1 or more years during their formal schooling. Of this 61%, 25% had 4 or more years of water conservation education, with one respondent receiving at least 8 years of formal schooling on the topic. This curriculum occurs heavily in early education and can manifest in several forms. This education can be direct, where teachers instruct students to save water on everyday tasks such as brushing teeth, or it can be more creative. Children from kindergarten through grade school are taught songs valuing water as a resource, with lyrics admonishing water loss, "it's a shame of every drop, don't waste water" (O. Ben Zeev, personal communication, June 30, 2019). Lyrics from a popular kindergarten song titled "a pity on any drop" are commonplace and emphasize the act of saving water so everyone can have equal access to it (O. Ben Zeev, personal communication, June 30, 2019). Schools further promote a water conscious culture in a myriad of methods and their effects extend beyond students. In the city of Efrat, Mayor Oded Revivi specifically utilizes young children to enact city wide change. "So first of all, generally speaking, kids are the best change agents. If I want to do something or educate my residents to recycle more things, I will always go to the lowest grade, the youngest kids in the education system. I have to invest in them the least amount of time and they can be my quickest, most effective change agents." In one instance, Mayor Revivi sent kindergarteners home with a clock that had the time and cost of water based activities. These activities included items such as the price of showering for 5 minutes, brushing teeth for 2 minutes, and flushing the toilet. Revivi aimed to instill ideas of water consciousness at a young age while also encouraging households to save water (O. Revivi, personal communication, June 23, 2019). Early water education in kindergarten holds a central theme in Israeli education and some teachers even go to great lengths to instill a fear of water scarcity or irresponsible water use. In the small desert town of Merhav Am, students recall teachers scaring them to save water with warnings that the water will run out if they waste it. Teachers would often tell kindergarteners the value of saving water and instill practices such as finishing water bottles when playing outside (O. Ben Zeev, personal communication, June 30, 2019). Water education, particularly for young students, takes on many forms and instills the need to save water.

Early formal education in water conservation is supplemented by frequent exposure to water topics through media. In the expert survey, respondents cited the media more than any other source as the largest contributor to their

knowledge on water conservation. The Kinneret, or Sea of Galilee, holds biblical importance and is associated with the survival of Israel. Every day, radios and news broadcasts in Israel announce the rise and fall of the water level of the Kinneret. Increases in water level by even 2 to 3 centimeters elicit positive emotions while decreases elicit negative ones. Dr. Gideon Oron described a common understanding, "We have a joke, if the Sea of Galilee is full of water, then people are happy" (G. Oron, personal communication, June 11, 2019). Israelis from adolescence to adulthood constantly hear about the water level and discussions surrounding it. The Kinneret, one of the nation's most visible water assets, has become an unofficial proxy for the level of water security within the nation (G. Oron, personal communication, June 11, 2019).

Television plays a major role in swaying public opinion and creating memorable impressions. In the late 2000's, the Israeli government launched a television-based campaign titled "Israel is drying out" to compel Israeli citizens to use less water. The most prominent ad featured supermodel Bar Rafaeli with her skin drying out as she discusses Israel's water issues. This single television ad was mentioned in multiple of our research interviews and a significant portion of Israelis recognize and remember the ad to this day (E. Av, personal communication, July 2, 2019; O. Ben Zeev, personal communication, June 30, 2019; J. Zvuloni, personal communication, June 17, 2019). The success of the campaign led the Israeli government to air another ad in 2014 starring Bar Rafaeli and warn that Israel still faces water shortages despite advances in desalination (E. Av, personal communication, July 2, 2019). Television campaigns have existed for decades and early campaigns in the 1970's through the 1990's had different approaches. Some served more as informative commercials on saving water, giving advice such as how to wash a car or when to water a garden (O. Ben Zeev, personal communication, June 30, 2019). Others had targeted audiences such as children. One had featured a young boy brushing his teeth and showering, with the child shouting a slogan at the end "in the meantime close the [faucet]" (E. Av, personal communication, July 2, 2019). The government also utilized other mediums such as the Internet and newspapers to communicate similar ideas; however the efficacy of these methods is unknown (N. Levy, personal communication, June 19, 2019). Public campaigns have also used other nontraditional mediums such as placing stickers in public restrooms advising to turn off the faucet (J. Zvuloni, personal communication, June 17, 2019). These campaigns are so pervasive and widespread that 96% of the expert survey respondents have seen an advertisement about water conservation.

Interpersonal interactions and personal experiences offer further socialization into water conservation values. Parents are often children's first introduction to water conservation. Parents teach their kids to save water from closing off faucets to teaching them songs about the value of one cup of water (O. Ben Zeev,

personal communication, June 30, 2019; J. Zvuloni, personal communication, June 17, 2019). In the expert survey, parents were the 2nd largest contributor to water conservation education, falling one respondent short of media. Israelis caution visiting foreigners to save water and when Israelis go abroad, they have been known to admonish others for wasting water, even in water abundant countries (N. Avieli, personal communication, June 26, 2019). Nearly 1/3rd of the expert survey respondents claim to feel judged when their peers perceive them to waste water. Israeli couples can even get into arguments over excessive water use (H. Ido, personal communications, June 24, 2019). Israelis self support a water conscious culture to effectively teach or pressure others to utilize water efficiently and conservatively. This culture is strong enough to elicit physical and emotional distress when Israelis see water loss. One sociologist describes the emotion, "I can not see something like this. I am angry when my kids waste water. I don't like it. I feel pain in my body when I see [it] because I was educated in this way" (N. Avieli, personal communication, June 26, 2019).

Religion, education, media, and interpersonal interactions produce a nation that highly values water as a resource. From birth to adulthood, Israelis are constantly barraged by messages to conserve water and this engenders a culture of water consciousness, therefore altering behavior. This consciousness manifests itself into practices, knowledge, and beliefs seen in the expert survey. The following statistics have all been derived from the expert survey. In terms of practices, a supermajority of respondents say their showers are less than 10 minutes, with 39% saying they are less than 5 minutes. On top of this, 69% say they only wash their car every other month or less, being less than 6 times per year. Of these, most say they only wash their car 3 times per year. In terms of knowledge, 68% correctly say farmers are the largest consumers of water and 81% are confident that they know and understand the source of their drinking water. Although Israel holds many global achievements for its efforts in water conservation and reuse, Israelis believe there remains much room for individual improvement. When asked how often they conserve water when presented the opportunity, only 27% said "every time." A majority of 58% said "most of the time." Despite Israel's significant advancement in water conservation, individuals still believe that more can be done. For further evidence, 89% of respondents said they can definitely do more to conserve water in their everyday lives. A more revealing facet: the majority of Israelis adamantly believe water is a scarce, finite resource. A supermajority of 73% of respondents believe that past generations used less water than we do today. The belief that there is an ever increasing amount of water usage is coupled with the belief that water is a depleting, finite resource. To add to this belief, 39% of respondents believe that in the next 25 years, in their area, there will be a "considerable shortage" of water. Overall, about 70% of respondents believe that there will not be enough water to meet demands in

that time frame. No respondents claimed that there will be "more than enough" water to meet future demands in that time frame. When asked about their concern for immediate water shortage, 39% of respondents claimed to be moderately worried (3 on a 1–5 scale), with 12% being very worried (5 on a 1–5 scale). Only 19% claimed to not be worried at all.

Government and Economics

While culture creates a self-perpetuating consciousness of water conservation, the government can provide the basis to enforce and guide such a culture. Water laws, pricing, and programs modify behavior to ultimately reduce water usage. Additionally, the Israeli government has taken proactive measures to steer public behavior and allocate funds to invest in water technologies.

Laws pertaining to the ownership of water allow the Israeli government to forcibly implement water saving practices. Israel's Parliament, the Knesset, passed the Water Law of 1959 which gave the government "widespread power to control and restrict the activities of individual water users in order to further and protect the public interest" (Siegel, 2017, p. 17). The Water Law of 1959 and the laws that led up to it effectively gave the government control over all water resources. The law granted the government control over water in privately owned lands and gave government ownership to every drop of rain. Unlike the US, where an individual may own the source of a river or aquifer if they own the surrounding land, Israel allows the government to maintain ownership of the mineral rights to any water source throughout the country. Making water a public resource allowed the Israeli government to make sweeping infrastructure changes, diverting water to much needed areas and using sources of water sustainably. This law has greatly shaped Israeli water usage as its legal precedence has given rise to many other water laws. In terms of directly limiting citizen's water usage, the government passed laws to mandate water saving technologies in consumer and commercial usage. In urban areas, the average Israeli consumes 100 liters of water a day and about half of this is used for toilet water. The government required the installation of dual flush toilets which offer a liquid waste flush option to decrease water usage per flush by half (J. Sack, personal communications, July 3, 2019). The same logic has been applied to water faucets. All water faucets in Israel are required to have aeration filters which have the potential of reducing water usage up to 70% (J. Zvuloni, personal communication, June 17, 2019). These types of laws offer an explanation as to why 57% of the expert survey respondents have appliances such washing machines or dishwashers that are water-efficient and why 88% use drip irrigation. Beyond technological mandates, the government also has laws concerning direct water usage,

such as preventing irrigation of grass in particularly dry areas or prohibiting car washes with hoses in times of drought.

Due to the widespread laws regarding water ownership, the government directly controls the pricing of water, and pricing coercively enforces water conservation. Mekorot is the national water utility company of Israel and controls the National Water Carrier, the main pipeline which distributes water throughout the nation to municipally-owned, local utilities or "water distributors." Mekorot, as a "national monopoly," must operate alongside the distributors under a legal mandate in which end users or consumers pay the same price nationwide. "All customers should pay the same tariff for water, regardless of where they are . . . Farmers in the north could have free water, [but] they still have to pay for the water they take, and by [doing] that, they subsidize the farmers down in the south, which are obviously paying much less than the real cost of the water [they receive] (H. Ido, personal communications, June 24, 2019). Israel is able to adopt this admittedly unique water supply chain due to its small geography and ability to use a single company, Mekorot, as a benchmark for pricing water nationwide. This end user price is determined by an independent government regulator, using Mekorot's projected national expenses, and remains fixed regardless of the particular transportation costs to supply a certain area.

Mekorot acts as a national, government-owned monopoly with the sole right to purchase water from the government and supplies water in bulk to the city and municipal utilities. The water distributors then in turn provide water to their citizens. This system works by having consumers pay local distributors a nationally set rate for water, and the distributors then pay Mekorot a percentage for the water their municipality consumes. This percentage changes depending on the municipality's access to water. For example, Tel Aviv, having easier access to water, might pay 90% of each shekel it receives from its consumers back to Mekorot, whereas Jerusalem might only pay 50% (H. Ido, personal communications, June 24, 2019). Without any government subsidies, these companies must act similar to private sector firms while also completing their legal mandates. The companies must manage their expenses and revenues within the narrow realm that the government provides between the wholesale cost of water and the end user purchase price. This encourages firms to diligently monitor their water usage and invest in water saving technologies.

In this system, citizens with easy access to water are paying more than the cost to receive it, and the price is substantial enough to discourage waste in areas with relatively plentiful water. This also prevents water from being prohibitively expensive in the desert and remote regions of Israel, encouraging agriculture and migration to these regions (H. Ido, personal communications, June 24, 2019). However, this true price of water brings with it dissatisfaction, with 50%

of expert survey respondents believing end users should not have to bear the full price of water. Furthermore, Israel uses block rate pricing, drastically increasing prices after a set allotment of water is used. On average, each person receives 1 to 2 cubic meters per month and each household has 8 to 10 cubic meters per month at the initial price (N. Avieli, personal communication, June 26, 2019). The initial price is typically 2 to 3 shekels per cubic meter. However, after the initial allotment, the next allotment could potentially be 5 shekels per cubic meter and then to 10 shekels after that, continually increasing (O. Ben Zeev, personal communication, June 30, 2019). This same style of pricing is applied to agriculture, albeit subsidized to roughly 1/3rd of the household water price, but the price still remains significant enough to be 5% to 7% of operating expenses. In moshavim, cooperative agricultural towns, the price and amount of the initial water allotment can be severe enough to cause the entire moshav to limit crop planting (A. Peleg, personal communications, July 3, 2019).

The government also engages in less forceful ways to ensure water conservation, providing incentives and support. The government funds and hosts water specific technology incubators which encourage entrepreneurial innovation. For two years, these incubators provide inventors financial, technical, and business support to produce a prototype of their invention. Once the prototype is completed, it is up to the inventor to then market, produce, and sell their invention. These inventors typically turn to venture capitalists or angel investors to start refinement and production of their invention. If the inventor is successful, the Israeli government support is seen as a loan and the costs are expected to be paid back. If the inventor fails, the support is seen as a grant. These incubators provide inventors a low risk and high reward system to produce water saving or generating technologies such as new water filtration systems. This process is further facilitated because the Israeli government hosts water technology conferences to share ideas and connect inventors to investors. In the early 2000's, Israel hosted the Water Technology and Environmental Control convention, gathering thousands of participants in its first year, and conferences have continued to be a forum to support Israeli inventors (J. Gilron, personal communications, June 20, 2019). Aside from technology, the government provides industry specific subsidies to reduce water usage. For example, in the water intensive fruit industry, the government provides further water subsidies if the growers did not exceed a set amount of water consumption. This has proven to reduce water usage by 35% to 40%, and this method rewards, rather than punishes, agricultural producers (J. Sack, personal communication, July 3, 2019). The Israeli government engages in a variety of unique methods to save water, operating with both a cost and reward system.

The firm approach by the Israeli government could potentially be interpreted as overbearing but according to the expert survey data, Israeli citizens expect

even more measures from their government. With 73% of respondents claiming to understand government plans to address future water needs, Israelis are aware of their government's efforts with water conservation and resources. With this awareness, 96% of respondents agree nationwide funding should be used to implement water conservation strategies. This knowledge coupled with this belief explains why Israelis are calling for more government effort. For reference, 46% of respondents said that they are not content with the Israeli government's current attempts to conserve water. A sizable 89% say their local water utility is not doing enough to educate the public on means to conserve water and 50% claim that they are not even aware of any water conservation efforts from their local government.

Further Discussion and Future Research

A primary point for further study is the longevity of this water consciousness. In the early 2000's, Israel raised the cost of water, in addition to the block rate pricing, to reflect the actual economic costs and for the first 10 years of implementation, water usage drastically lowered. However, the last few years have shown an overall increase in water consumption per capita, indicating households are becoming accustomed to higher water bills and are therefore using more water (H. Ido, personal communications, June 24, 2019). To add to this concern, some Israelis have seemingly adopted a penchant for the American lifestyle with a tendency to cultivate water intensive grass lawns and to have A-frame roofs suited for heavy snow/rain, an unnecessary use of resources considering many of these houses are in the desert (N. Avieli, personal communication, June 26, 2019). Furthermore, the addition of desalination plants as a novel source of water has initiated a cultural shift in which Israelis see water as plentiful and no longer scarce, despite existing needs to continue water conservation (N. Levy, personal communication, June 19, 2019). To reference a previous statistic, 19% of the expert survey respondents are not worried at all about an immediate water shortage, and this percentage has the potential to grow. Israel's excellent capacity to recycle and desalinate water were not talked about in length in this discussion, but their effects on future behavior could end Israel's headway as the global leader on water conservation.

Conclusion

By pairing a self-supporting culture to centralized pricing and governance, Israel has developed an effective strategy to encourage water conservation. The nation

has utilized these methods to achieve and maintain its position as the global leader in water conservation and reuse for decades. Israel's approach to encouraging water conservation is a valuable lesson to policymakers and organizations aiming to promote water sustainability. Two methods in particular could benefit decision makers because of their effectiveness and replicability: water pricing and cultural exposure. From the research interviews, over half of interviewees noted that water pricing is a key strategy in influencing water conservation. This strategy is largely the most visible and impactful to consumers. However, appropriate water pricing does not have to stem from a national monopoly system as it does in Israel. This strategy can be achieved through initiatives at the local water utility level, as has been done in El Paso. In the dry, Texas town, the local utility works in unison with the local government to enforce a series of penalties and rewards surrounding water use (Schlanger, 2018). In addition to water pricing, 30% of expert survey respondents cited the media as their primary channel for learning about water conservation. Following this statistic, 27% of expert respondents cited their parents as their primary educator on water conservation and 23% cited their friends. Lastly, 15% of expert respondents cited education as their primary means for learning about water conservation. These statistics indicate a cyclical culture in which water consciousness permeates generations of society. Government or NGO-backed initiatives showcase water conservation in the media and work alongside formal schooling on the topic to educate the populace. When publicized in meaningful and appropriate volumes, the initiatives develop a water conscious culture which transcends from generation to generation. These two strategies, centralized pricing and cultural exposure, form a powerful combination which influences both the economic use and the societal value of water.

However, these strategies are unlikely to come about in many countries unless the incentive exists for these countries to adopt these strategies. Legal reforms require both economic and political capital, and the high price of adopting a coherent water conservation strategy will be too much for many countries without an urgent need arising. Israel has faced this need for decades as a country located in an arid climate with a continuing threat of desertification. Similar communities such as West Texas or the Southwestern United States might find themselves facing this threat and could be incentivized to follow Israel's lead in water conservation. El Paso has already begun this process by investing in desalination and purification technology, promoting water conservation exposure through the media, and enforcing local ordinances to conserve water. One such program adopted by the El Paso Water Utility is 'Willie the Water Drop,' a city mascot who visits schools and teaches children about the importance of saving water (Schlanger, 2018). Experts from Israel facilitate much of this technological and cultural exchange.

This research has identified several major themes which communities or countries might use to encourage water conservation, however, much of our findings remain qualitative and anecdotal. Our interviews utilize the experience and knowledge of a select group of experts and professionals in the field of Israeli water conservation, but these individuals are not enough to statistically represent the entire nation of Israel or a cohesive opinion of water professionals globally. In order to further solidify which methods are most effective in terms of encouraging water conservation, further quantitative research is needed. Still, these findings represent the qualitative history and attitude of many in the nation of Israel and provide a thorough look into the consciousness surrounding Israeli water conservation.

In conclusion, Israel has created a culture and societal consciousness which preserves and conserves water resources, modifying behavior directly or indirectly through religion, education, media, and interpersonal interactions. The nation has further advanced water conservation by establishing laws requiring the protection and conservation of the asset, pricing that discourages water use, and programs to develop the next generation of water saving technology. However, the same technologies and practices which have allowed Israel to become a global leader in water conservation could very well lead the country to increase its water usage and diminish its cultural focus on water conservation. These are all important lessons for future policymakers to keep in mind when embarking on new water conservation initiatives.

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The East Asia Crisis – Fundamental Indicators and the Need for Bailout Intervention

DANIEL MOTOC

The East Asia Crisis that began in 1997 was unique in its ability to cripple countries that had previously enjoyed years of unprecedented economic success. This paper provides a brief review of the East Asia Crisis and explores differences between countries that required bailouts and those that did not. The results show that the countries that required bailouts were characterized by greater current account deficits, higher levels of domestic credit to the private sector, lower export shares of GDP, and more volatile debt composition than those less affected. Meanwhile, the groups were similar in their export growth and inflation rates. These findings could inform policy measures aimed at preventing and managing future financial crises.

1. Introduction

The East Asia Crisis began on July 2, 1997 with the devaluation of the Thai baht and quickly spread to neighboring countries. The East Asian countries had experienced years of robust economic growth leading up to the crisis, leading many economists and policy makers to try and identify the causes of the crisis. Potential explanations of the crisis include weak fundamentals, investor panic and improper policy responses, weaknesses in the financial sector, or some combination of the above. In this paper, I assess the importance of economic fundamentals as a cause of the crisis by comparing current account balances, domestic credit to the private sector, exports, inflation, debt levels and capital flows between the countries that required bailouts and those that did not.

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Relevant Literature

Numerous theoretical and empirical papers have been published trying to understand how financial crises unfold. Some of the plausible theories that have emerged analyze business cycles and imperfections in the credit market, flaws in exchange rate regimes, and information asymmetries as possible root causes of these crises. On the other hand, empirical studies have analyzed the relationship between economic fundamentals and financial crises, the timing of speculative runs on currency, and the spread of contagion.

The existing literature has argued that in most crises, it is possible to find a set of fundamentals that could explain which countries would be most affected and to what extent. Kaminsky, Lizondo, and Reinhart (1998) construct an index of warning indicators consisting of exports, GDP, real exchange rate deviations, inflation and several other variables and analyze the extent to which the index predicts a currency crisis (1). Sachs, Tornell and Velasco (1996) present a model in which real exchange rate appreciation, lending booms and low reserves explain why some emerging markets were hit by crises following the peso devaluation in 1995 while others were not (2). Other literature has linked financial crises to fundamentals within the financial sector in particular. For examples of this, see Calvo (1995) and Velasco (1987) (3,4).

The literature on the East Asia Crisis in particular shows that the countries most severely affected ran larger current account deficits, showed greater exchange rate volatility and had higher indicators of financial and real instability. For example, see Corsetti, Pesenti, and Roubini (1998) (5). Other researchers have blamed the international organizations who pushed the East Asian economies towards rapid financial deregulation, leading to large capital inflows that quickly reversed upon the devaluation of the Thai baht. For examples, see Dissanaike and Markar (2009) and Austin (2009)(6,7). Finally, another explanation for the crisis is that trade linkages and in particular export policies are a determining factor in the magnitude of currency crises and played a key role in the East Asia Crisis. For example, see Khan (2018) (8).

The main contribution of this paper is to the literature that studies the role of fundamentals in financial crises. I provide a summary of the similarities and differences between 11 East Asian economies in several fundamental variables in order to differentiate which fundamentals may have been relevant in determining the severity of the East Asia Crisis. I also use a different methodological approach in which I compare countries that required IMF bailouts with those that did not, lending additional support to the literature arguing that large current account deficits and excessive credit levels make countries susceptible to crises. The second main contribution of this paper is to the literature analyzing the effect of exports and trade on the crisis. The data in this paper

raise doubt on the relevance of export levels during the East Asia Crisis, despite the significant decrease in exports following the reversal of the Plaza Accord. The data shows that the countries requiring bailouts had lower reductions in their export revenues on average, suggesting that export levels alone likely did not cause this crisis. Rather, it may have been the interaction between debt and export levels that proved relevant, and future research should analyze the mechanism through which the two interact in order to guide more effective policy design.

The rest of the paper is organized as follows. Section 2 discusses the methodology. Section 3 lays out a brief overview of the build up to the crisis. Section 4 presents the data on fundamentals. I conclude in section 5.

2. Methodology

The sample used in this paper includes the 10 members of the Association of Southeast Asian Nations (ASEAN) and South Korea. Although there are multiple metrics to determine how severely a country is hurt by a financial crisis, for the purposes of this paper the sample was split into two groups: countries that required IMF bailout packages (Thailand, South Korea, Philippines, and Indonesia (TIKP)) and those that did not (Brunei, Cambodia, Laos, Malaysia, Myanmar, Singapore and Vietnam (non-TIKP)). This is a similar empirical approach to that used in *Rebooting the Eurozone: Agreeing a Crisis Narrative*, a policy research paper analyzing the Eurozone Crisis from the Center for Economic Policy Research.

The two groups are comparable for two reasons. First, by the beginning of the crisis, Indonesia, Malaysia, Singapore, the Philippines, Thailand, Brunei, Vietnam, Laos and Myanmar were all part of the ASEAN. Part of the ASEAN's mission was to facilitate economic growth and increase collaboration between the countries on agriculture, industry, and trade (8). As part of this economic union, the countries shared common economic policies. Second, the countries in the two groups had similar savings rates, GDP growth rates, and exchange rate regimes leading up to the crisis (see appendix and (9)). With the exception of the Philippines which operated an independently floating exchange rate, all other countries in the sample used a managed float or direct peg system.

All data used in this paper were collected from three sources – the World Bank Online Database, Penn World Table, and the Bloomberg Terminal. In section 3, I use stock price data collected from the Bloomberg Terminal while all growth-accounting data was collected from the Penn World table. The data in section 4 on fundamentals were taken from the World Bank's *World Development Indicators* database. In this section, I look at the current account balance,

domestic credit to the private sector, export levels, inflation, short-term debt, and foreign direct investment in the sample countries and plot a non-weighted average for the two groups between 1990 and 1997 for each variable.

3. Buildup to the Crisis

Beginning in the 1960s, the East Asian countries experienced tremendous economic growth. A World Bank Report from 1993 credited much of this growth to low taxes, improved government institutions, and export-oriented policies. Four countries in particular - Hong Kong, Singapore, South Korea, and Taiwan - became known as the "Four Asian Tigers" due to their exceptional growth rates (over 7% per year), rapid industrialization, and their eventual development into high-income economies in the 21st century (10).

Meanwhile, the East Asian countries saw a significant increase in capital inflows. One such type of capital flow, foreign direct investment (FDI), occurs whenever a firm or investor from one country establishes a business or acquires an existing entity in a foreign nation. In the second half of the 20th century, firms began to invest in the region, contributing to the rapid economic growth. The following figure illustrates net foreign direct investment (FDI) inflows to the East Asian economies, which soared 3,000% between 1970 and 1990.

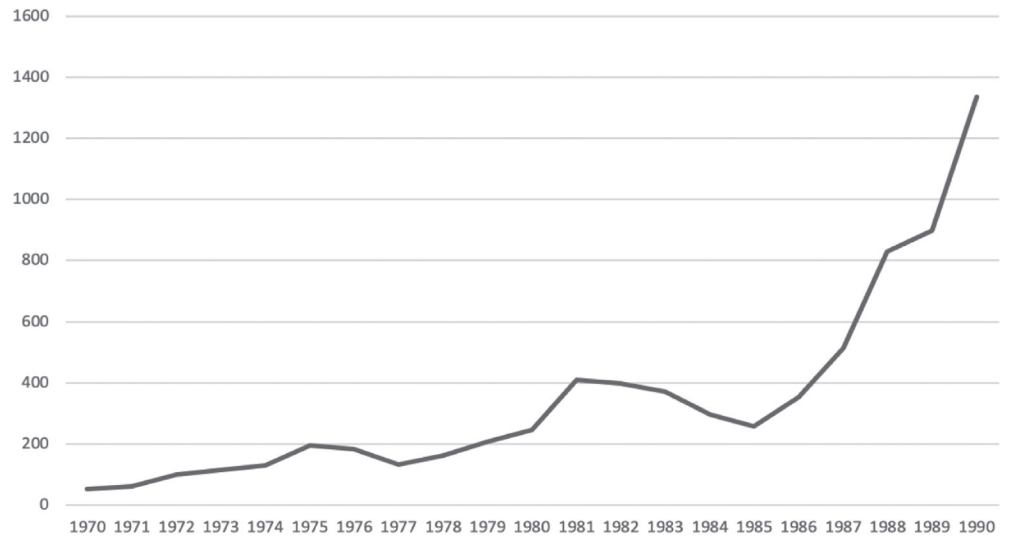

Figure 1: Foreign Direct Investment, ASEAN Member Nations and South Korea (current USD, millions)
(Source: World Development Indicators, the World Bank Group)

A natural next question is, how much of the growth in real GDP in East Asia over this time period could be accounted for by growth in the capital stock? To answer this question, I conducted growth-accounting using a Cobb-Douglas production function with constant returns to scale of the form $F(K,L) = AK^{\alpha}L^{1-\alpha}$, where A is a parameter representing technology, K stands for capital, L stands for labor, and alpha represents capital's share of income. Taking natural logs and subtracting, the production function can be rewritten in terms of growth rates:

$$\gamma_{Y,t} = \gamma_{A,t} + \alpha\gamma_{K,t} + (1-\alpha)\gamma_{L,t}$$

where gamma represents the corresponding growth rate in year t. The growth in labor was approximated by the growth in population and the growth in technology was approximated by the growth in total factor productivity (tfp). The results for the ASEAN nations and G7 nations are shown below.

Thailand	Indonesia	Korea	Philippines	Singapore	Malaysia	Laos
117.75%	46.74%	50.69%	84.81%	97.88%	87.83%	63.48%

Capital Contribution to Output, G7 Nations (1980–1997)

United States	United Kingdom	Germany	France	Italy	Japan	Canada
30.09%	51.26%	29.51%	20.89%	61.02%	49.25%	43.28%

(All data collected from Penn World Tables)
Note: Full tables can be found in the Tables section

Figure 2: Capital Contribution to Output, ASEAN Nations (1980–1997)

As Figure 2 shows, capital contributed more to output in the ASEAN nations than in the G7 nations between 1980 and 1997. Capital was much more important to these countries' growth than in the G7 nations, and this is one potential explanation of why the subsequent reversal of capital flows led to such a severe contraction. In Singapore almost 100% of the growth in real GDP could be explained by growth in the capital stock. In Thailand, the capital stock grew even *faster* than real GDP.

Although this analysis only factors in real capital, East Asia was also experiencing large inflows of financial capital. Domestic credit and stock markets boomed: in the 10 years leading up to the crisis, the Indonesian stock market as

measured by the JCI index rose 682%, while the Philippines and Malaysian indices rose 321% and 198% respectively. Thailand and South Korea showed more modest growth of 94% and 34% respectively (stock price data collected from the Bloomberg Terminal).

These countries enjoyed tremendous success during the second half of the 20th century, but the reversal of the Plaza Accord in 1995 brought financial difficulties to the Asian economies (11). The United States agreed to let the US dollar appreciate against the yen and Deutsche mark, which made German and Japanese exports more competitive with Asian exports (11). Furthermore, the stronger dollar made it difficult for Asian economies to borrow in dollars, and it became increasingly difficult for them to subsidize their industries (11). Foreign reserves were being depleted, and investors questioned whether these countries could support their currency pegs. On May 14, 1997, there was a large speculative attack on the Thai baht. Prime Minister Chavalit Yongchaiyudh promised he would not devalue the baht, but Thailand eventually ran out of reserves and was forced to float the baht on July 2 (11). This marked the beginning of the East Asia Crisis, as capital flight ensued and contagion spread throughout the region.

A natural question in policy circles is whether the outcome of the crisis could have been predicted. The next section explores this question by comparing the countries most severely affected by the crisis with those more mildly affected and seeing if there are common economic fundamentals among them.

4. Fundamentals

Current Account Balance

The current account is a measure of a country's trade balance plus its net income and transfer payments from abroad. The current account also measures net capital flows and reflects whether a country is a net borrower or net saver. A current account deficit means a country is a net borrower while a surplus indicates it is a net lender. Figure 3 shows the average current account balance of the TIKP and non-TIKP nations in the years leading up to the crisis.

The graph shows that, on average, the TIKP nations ran greater current account deficits compared with non-TIKP nations in the decade before the crisis, but the two groups converged as the crisis hit. Non-TIKP nations saw large inflows of capital beginning in 1992, and both groups experienced sudden stops when the capital flows reversed in 1996. As the graph indicates, the contraction was more severe for the TIKP countries (2%) compared with non-TIKP countries (~0.5%).

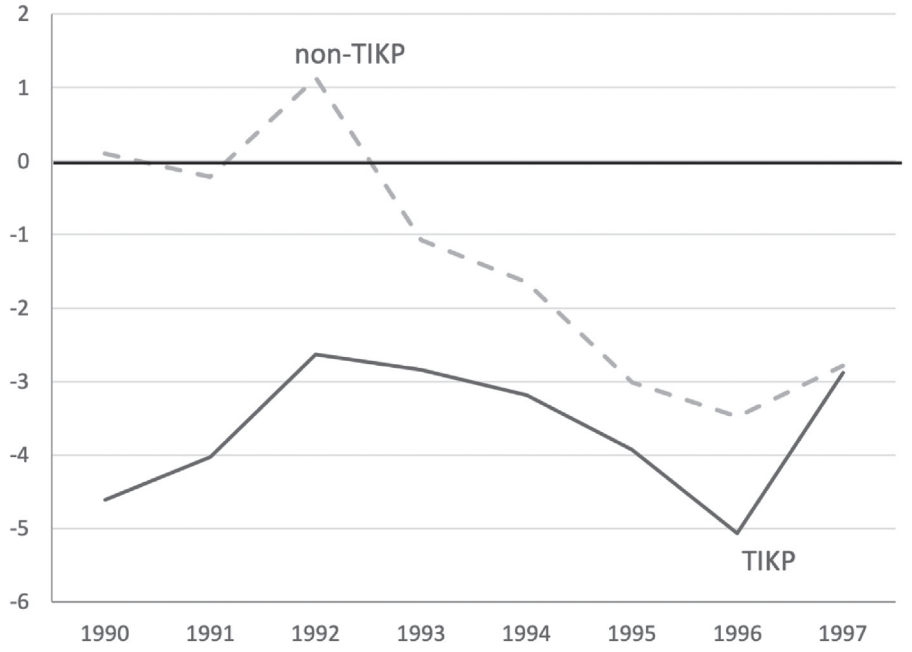

Figure 3: Current Account Balance (% of GDP)
(*Source*: World Development Indicators, the World Bank Group)

Domestic Credit to Private Sector

High GDP growth rates often accompany credit booms. Large expansions in credit facilitate spending and thereby increase GDP, especially if credit is used to finance investment in productive assets. However, credit may also lead to high debt levels and can adversely affect the economy if directed towards less productive resources. Credit booms generally accompany the expansionary phase of business cycles, with rising GDP, rising inflation and falling unemployment. Contractions in credit tend to accompany the contractionary phase of business cycles, with declining GDP and increasing unemployment. Figure 4 looks at domestic credit to the private sector as a share of GDP for the two groups. The TIKP countries saw their private sectors borrowing more, with a larger expansion in credit.

Exports

Some economists propose that the export-oriented policies mentioned earlier were to blame for the crisis (for example, see (12)). The US dollar depreciated after

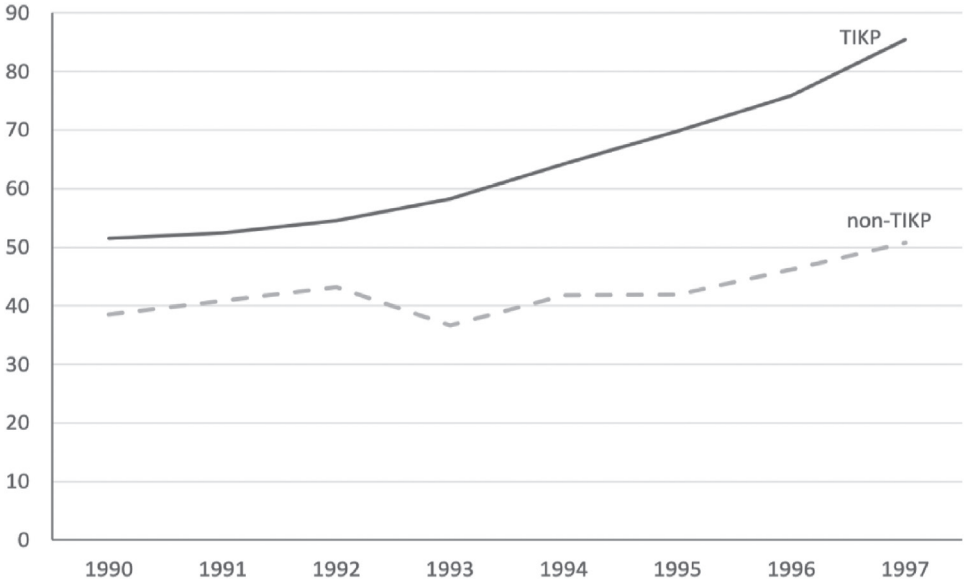

Figure 4: Domestic Credit to Private Sector (% of GDP)
(*Source*: World Development Indicators, the World Bank Group)

the reversal of the Plaza Accord in 1995, which suddenly made East Asian exports more expensive and less competitive in global markets. According to this view, the subsequent collapse in export revenues was one cause of the coming crisis.

As can be seen in figure 5, the TIKP and non-TIKP nations had similar trends in export growth. Both groups experienced a marked decline in their export growth in 1995, but the TIKP nations saw an increase in their export growth in the year leading up to the crisis while the non-TIKP nations saw their export growth fall further. Figure 6 plots exports as a share of GDP for the two groups and shows that exports in the non-TIKP group accounted for almost twice as much of GDP as in the TIKP group. Even though in the non-TIKP group export growth fell further and exports contributed more to GDP, it was the TIKP nations that required bailouts. Thus, the data suggest that export levels alone were not to blame for the crisis; an interesting follow-up question would be to analyze the interaction between a collapse in exports and existing debt levels – it is plausible that a collapse in exports has a more detrimental impact on economies with high debt burdens.

Inflation

Inflation was relatively well managed in the ASEAN countries before the crisis. Laos was the only country with a hyperinflation episode, with an annual inflation

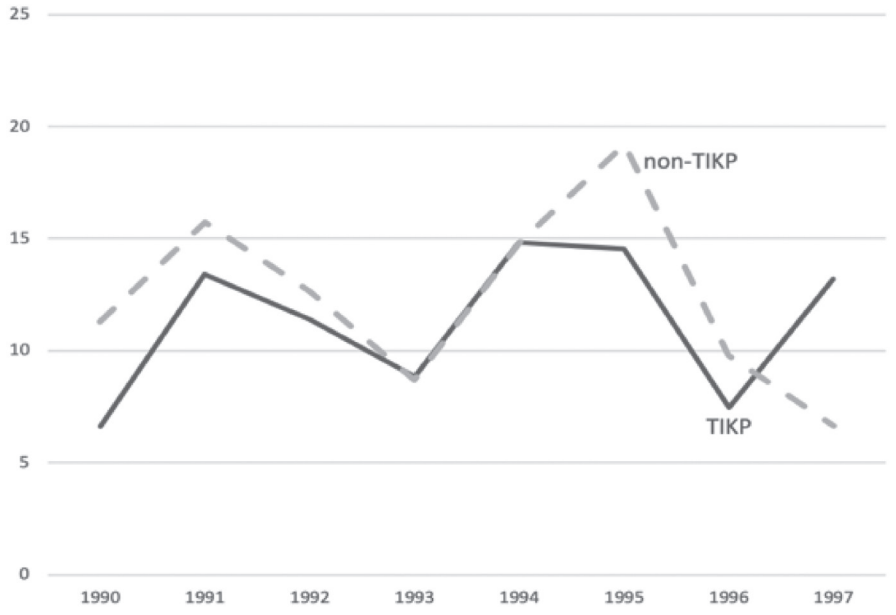

Figure 5: Export Growth (annual %)

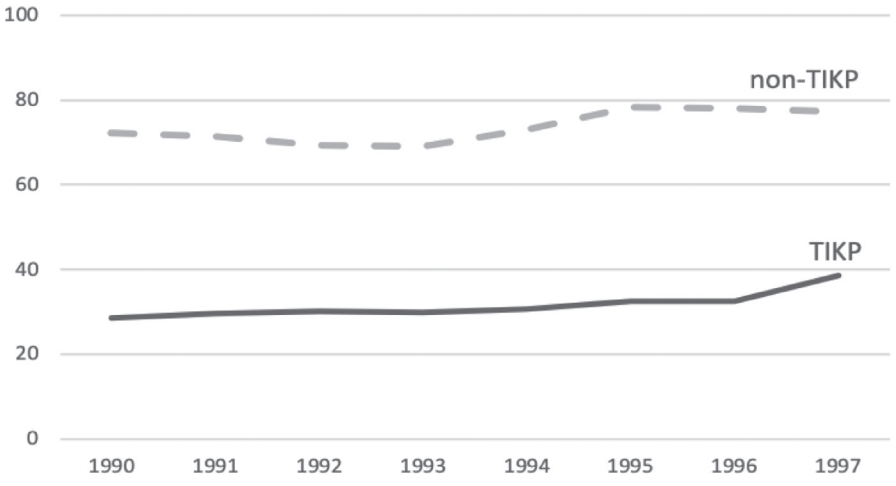

Figure 6: Exports (% of GDP)
(*Source*: World Development Indicators, the World Bank Group)

rate of 61% in 1989. However by 1993, its inflation rate dropped to 6%, and the average inflation rate from 1990–1997 for the Asian economies excluding Laos was 7.5%. Overall, Figure 7 shows no significant difference in the average inflation levels between the two groups from 1990–1996, but the two groups diverged

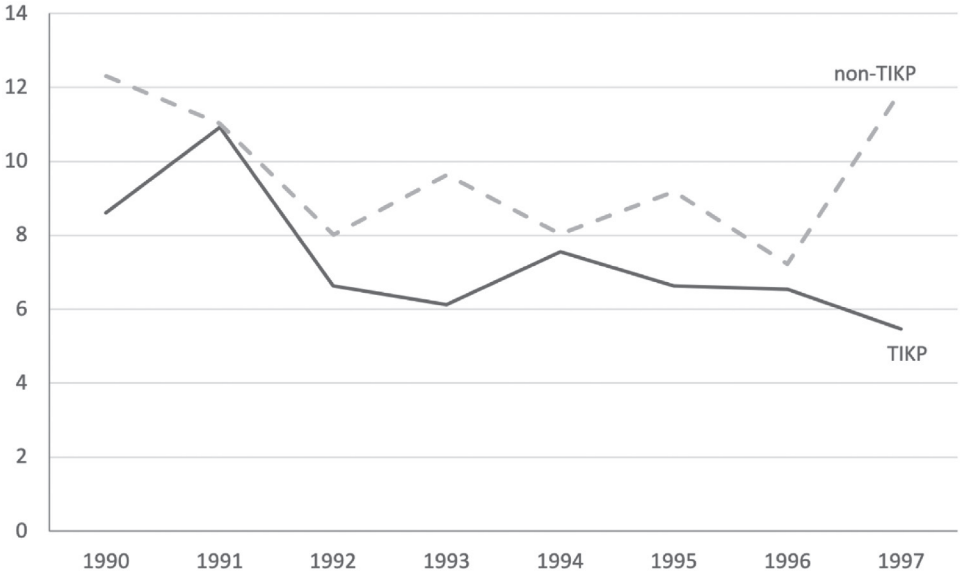

Figure 7: Inflation (annual %)
(*Source*: World Development Indicators, the World Bank Group)

immediately before the crisis. A closer look at the data reveals that in 1997, Laos and Myanmar saw their inflation rates nearly double, while inflation in the other non-TIKP countries remained roughly constant.

Debt

Debt is an important aspect of any crisis episode because even if a financial crisis is not a debt crisis at its core, it may quickly develop into one. During a sudden stop episode, countries desperate to borrow have to face higher interest rates. Higher interest payments on short-term obligations may be hard to meet, and lenders may be unwilling to rollover the debt. Faced with liquidity issues, banks must turn to a lender of last resort – usually their central bank. As the crisis unfolds, the government often buys out the debts of private banks. This is the famous "doom loop", and in this way, sudden stop crises may develop into debt crises (13).

Figure 8 looks at short-term debt in the East Asian economies in the period leading up to the crisis.

Since 1980, the debt stocks of the TIKP nations had much higher proportions of short-term debt on average than the non-TIKP nations. When the crisis hit in 1997, the TIKP group had six times more short-term debt relative to total debt than the other Asian economies had.

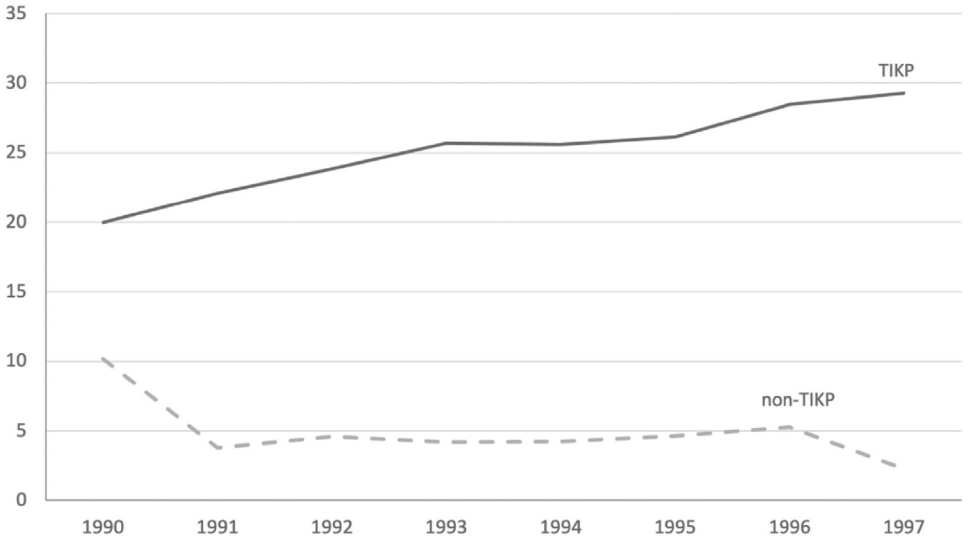

Figure 8: Short-Term Debt (% of External Debt Stock)
(*Source*: World Development Indicators, the World Bank Group)

The data seems to lend support to the traditional view among policy circles that short-term debt makes countries particularly susceptible to financial crises. However, some of the academic literature has argued that this is a case of reverse causality – financial institutions that want to provide liquidity to borrowers with poor credit ratings must borrow short-term, and it is the decreasing credit worthiness of debtors rather than the short-term maturity structure of debt that causes susceptibility to crises (for example, see Diamond and Rajan (2000)). I refrain from offering a causal relationship between short-term debt and the trajectory of the TIKP nations, and merely document the empirical relationship.

Foreign Direct Investment

As documented earlier, the Asian economies saw an explosion in FDI in the second half of the 20th century. Despite rapid FDI growth being common to the whole region, there were still notable differences in the composition of capital flows for the two groups. On average, the TIKP nations had much lower levels of net FDI inflows between 1990 and 1997 than the non-TIKP nations.

FDI is considered to be a longer-term, more stable form of capital flow as compared with other forms of capital. Since the TIKP nations had larger current account deficits but smaller inflows of FDI,capital flowing into the TIKP nations was likely more short-term and volatile in nature. The data also seems to be consistent with the view that short-term, volatile capital increases the fragility

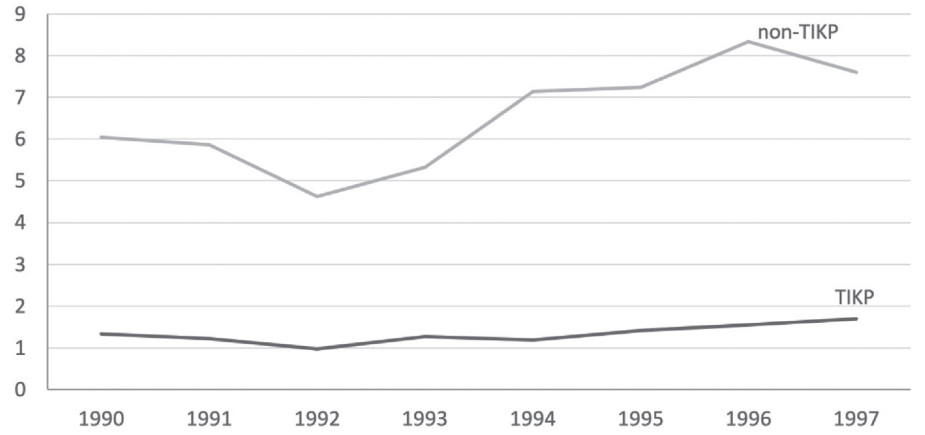

Figure 9: Foreign Direct Investment, Net Inflows (% of GDP)
(*Source*: World Development Indicators, the World Bank Group)

of a country's financial sectors and makes them more susceptible to a crisis. As explained previously, I merely document the relationship between volatile capital flows and the TIKP group membership and refrain from making any causal statements.

5. Conclusion

This paper provided a brief review of the East Asia Crisis and showed that a large part of economic growth in the Asian economies during the second half of the 20th century may be associated with growth in the capital stock. The paper then evaluated differences in several fundamental indicators (current account balance, domestic credit to the private sector, export levels, inflation, debt composition and FDI) between countries that required bailouts (Thailand, Indonesia, South Korea, and the Philippines) and countries that did not (Brunei, Cambodia, Laos, Malaysia, Myanmar, Singapore and Vietnam). The results show that the countries that were bailed out borrowed more, as evidenced by higher current account deficits and larger levels of domestic credit. Moreover, their aggregate debt stocks had a higher share of short-term debt, and their capital flows were more short-term and volatile in nature.

The data in this paper also raise new questions about the effects of trade policy in financial crises. The countries that experienced the largest collapses in export revenues and in which exports made up the largest proportion of GDP were not the countries that required bailouts, however they did have lower debt

levels. This suggests that perhaps the effect of international trade on a country during a financial crisis depends on other fundamentals, such as debt levels. This mechanism should be analyzed in future research so that policymakers can more effectively anticipate the economic impacts of trade reductions. Finally, inflation rates between the two groups were similar.

One potential limitation of this analysis is the lack of consistent economic reporting across these countries during the 1980's and 1990's. Several countries did not have full detailed data over this time period, and consistent detailed data would have allowed for a more accurate analysis. Furthermore, future research should evaluate whether these findings still apply if the sample is divided according to other metrics – the main alternative being contraction in GDP. Finally, future policy work should continue analyzing the extent to which economic fundamentals predict financial crises with the aim of developing policy tools to prevent or mitigate crises in the future.

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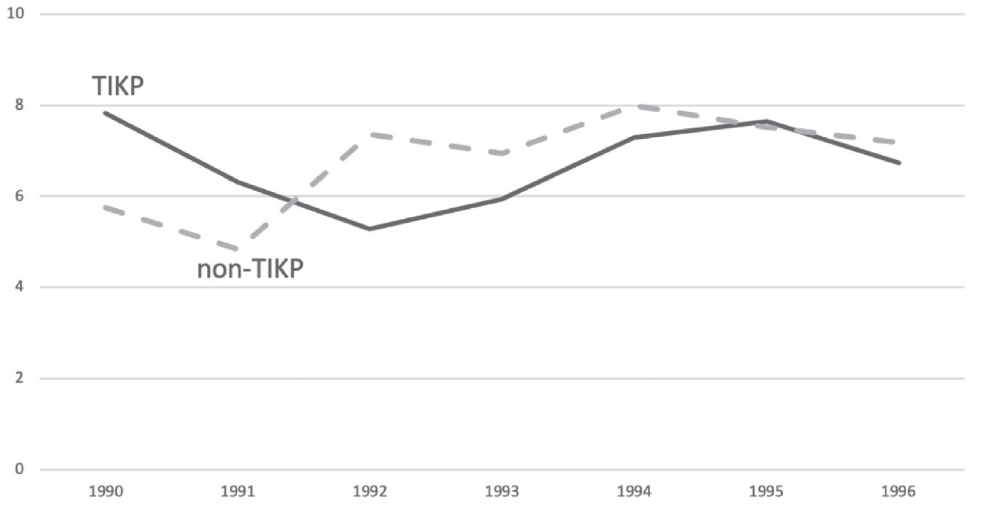
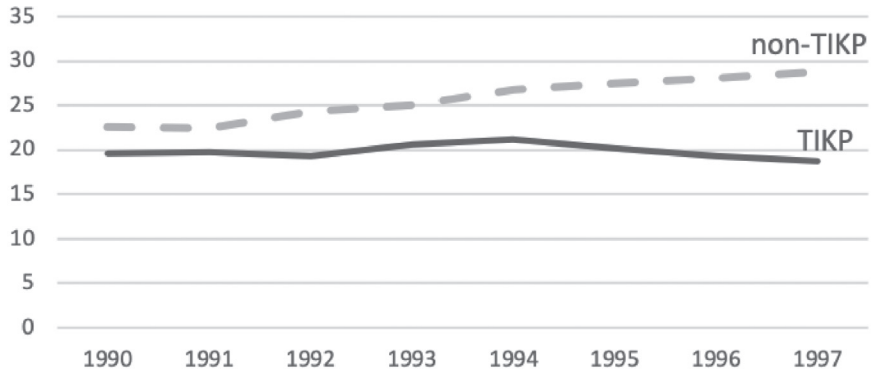
Appendix

GDP Growth Rates



Savings Rates



Tables

	Thailand	Indonesia	Korea	Philippines	Singapore	Malaysia	Laos
$\gamma_{Y,t}$	6.95%	6.35%	10.51%	5.23%	4.56%	6.72%	7.08%
$\gamma_{A,t}$	0.206%	0.539%	2.27%	–1.5%	0.309%	.094%	–5.43%
$\gamma_{K,t}$	14.11%	5.35%	12.56%	7.89%	7.98%	8.49%	7.76%
$\gamma_{L,t}$	1.48%	1.89%	1.15%	2.57%	2.46%	2.66%	2.62%
α	58.07%	55.52%	42.39%	56.18%	55.95%	69.50%	57.90%
Total Capital Contribution	117.75%	46.74%	50.69%	84.81%	97.88%	87.83%	63.48%

Table 1: *Solow Model Data – Asian Economies*

	United States	United Kingdom	Germany	France	Italy	Japan	Canada
$\gamma_{Y,t}$	3.39%	2.62%	2.87%	1.97%	2.54%	3.09%	3.04%
$\gamma_{A,t}$	0.87%	–0.10%	1.19%	–0.03%	–0.98%	0.13%	–0.47%
$\gamma_{K,t}$	2.64%	3.04%	2.55%	1.17%	3.55%	3.79%	4.23%
$\gamma_{L,t}$	1.02%	0.23%	0.21%	0.52%	0.07%	0.41%	1.14%
α	38.67%	44.30%	33.15%	35.35%	43.69%	40.16%	31.07%
Total Capital Contribution	30.09%	51.26%	29.51%	20.89%	61.02%	49.25%	43.28%

Table 2: *Solow Model Data – G7 Nations*

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Depression, Anxiety, and Pubertal Timing

Current Research and Future Directions

ABIGAIL G. RICHBURG, DOMINIC P. KELLY,
AND PAMELA E. DAVIS-KEAN

The current review synthesizes the literature regarding the effects of early and late pubertal timing on adolescent depression and anxiety. Early pubertal timing has been consistently shown to increase the risk of depression and anxiety in adolescents, particularly adolescent girls. Late pubertal timing has yielded more mixed results, with some research suggesting an increased risk of depression and anxiety in adolescent boys, indicating the need for future research specifically examining potential late pubertal timing effects on adolescent mental health disorders. Future research should examine the interactions between pubertal timing and pubertal status, as well as focus on diversifying samples to explore the pubertal experiences of many different groups, such as adolescent boys, low-SES adolescents, racial and ethnic minorities, and LGBTQ+ adolescents.

Keywords
pubertal timing, early pubertal timing, adolescence

Introduction

Throughout recent decades, the rate of child and adolescent mental disorder diagnoses has risen significantly.[1] A 2015 meta-analysis evaluating the prevalence of child and adolescent mental disorders in 27 countries from across the world estimated that 13.4% of children and adolescents are affected by mental disorders.[2] A 2019 analysis of the National Survey of Children's Health, a nationally representative sample from the United States, found that among children between the ages of 3–17 years, 3.2% and 7.1% had current depression and anxiety problems, respectively.[3] Although these disorders can occur in children,

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they often first appear during adolescence, and adolescents are at heightened risk for diagnosis.[4–8]

Some theories suggest that adolescence may be a risky period for mental health disorders due to the varied effects of puberty on development. Although puberty itself is universal biological transition, the experience of puberty can differ dramatically between individuals depending on a complex interplay between multiple biological and social factors.[9] One such factor is the effect of pubertal timing, defined here as the timing of the onset of puberty in relation to the timing of puberty for same-gender and same-age peers.[5, 10–11] Pubertal timing has been thought to affect the prevalence of depression and anxiety in adolescents.[12] Given that we know that the age of onset of puberty is decreasing in both boys and girls worldwide,[13–15] understanding the effects of earlier pubertal timing on mental health outcomes is urgent.

Due to the important consequences of mental health disorders during adolescence and their downstream effects across the lifespan, greater comprehension is needed of the effects of pubertal timing on the development of depression and anxiety. Therefore, this review serves two purposes: first, to examine the existing research on the relationship between early and late pubertal timing on adolescent depression and anxiety; and second, to identify the unanswered questions and future directions required for this crucial line of research.

Mental Health Disorders and Adolescence

Globally, depression and anxiety are the most common mental health disorders in total.[4, 16–17] Depression affects many facets of a person's life, including, but not limited to, daily functioning, mood, interests, appetite, cognition, and energy levels.[18–20] Likewise, anxiety has been shown to affect concentration, sleep, mood, and academic performance.[21–23] In general, depression is understood to be a collection of symptoms related to negative affect or an inability to feel pleasure,[6, 18, 24] while anxiety is understood to be a state of negative emotional arousal with an intense focus on the future.[23, 25] Theoretical work on depression and anxiety suggests that both disorders involve issues with downregulation of negative affect, but depression also involves issues with regulation of positive affect.[7, 26] Approximately 6% and 4–7% of adults worldwide suffer from Major Depressive Disorder and Generalized Anxiety Disorder, respectively.[19, 23] However, the two disorders frequently co-occur, and their symptoms often overlap.[27] Furthermore, it is important to note that many people experience symptoms of depression and anxiety without being diagnosed with either condition.[20–21] Data suggest that female depression and anxiety rates are higher than male depression rates throughout the lifespan.[28–30]

For adolescents, depression and anxiety are the most prevalent mental health disorders.[31–33] While mental health disorders can occur in children, they often first appear during adolescence.[4, 6, 8] Depressive symptoms are related to a range of potential problem behaviors during adolescence, including drug and alcohol use,[34–35] risky sexual behaviors,[36–37] and suicidal behaviors.[6, 38–39] Likewise, anxiety may lead to harmful outcomes during adolescence, including worse academic achievement,[22, 43] less peer acceptance,[44–45] more peer victimization,[44–46] and the development of depressive symptoms.[47] Adolescent depression has also been shown to negatively impact downstream outcomes in adulthood, including impaired educational attainment,[40] difficulty with interpersonal relationships,[31, 41] substance abuse,[31] and mental health issues throughout the lifespan.[31, 42] Similarly, adolescents who experience anxiety may experience downstream effects of their condition in adulthood, including decreased life satisfaction,[48] poor family relationships,[49] worse employment and income outcomes,[50] and mental health issues.[49, 51, 52]

Mental Health Disorders and Pubertal Timing

Puberty marks a time of significant biological and social maturation[9, 53] Biological maturation refers to hormonal changes, such as increased testosterone and estrogen, and resulting physical changes in the body, such as deepened voice for males and development of breasts in females.[54] Social maturation refers to the fact that puberty marks a social milestone that is influenced by gender-specific societal views and expectations.[55] The pubertal transition takes place throughout three to four years and involves complex interplay between these biological and social factors.[9] Globally, data suggest that the onset of puberty is becoming earlier.[13]

The timing of a child's pubertal onset is the result of biological and environmental factors and the interplay between them. Pubertal timing is a highly heritable trait;[57–59] an estimated 50–80% of the variation in pubertal timing is thought to be genetically determined.[57] Some environmental factors thought to affect pubertal timing include chronic psychological stress, father absence, and parental conflict.[55, 60–61] Evidence indicates that children who live in low socioeconomic status (SES) households, particularly girls, experience earlier pubertal timing on average than their peers who live in middle or high SES households.[55, 62, 63] Furthermore, pubertal timing seems to be influenced by childhood diet.[64–66] Various facets of childhood diets may differentially impact pubertal timing in boys and girls; for example, research has linked higher vegetable and lean protein consumption at the age of three to delayed breast development in girls and higher processed meat and refined grain consumption at the age of three to advanced testicular development in boys.[65] Inversely, higher processed and high-fat food consumption tends to trigger the onset of puberty earlier, particularly for girls.[66]

Another cause is speculated to be exposure to endocrine-disrupting chemicals that are often found in plastics and other materials.[14, 56]

The consequences of offset pubertal timing are broad; for example, earlier pubertal timing has been associated with worsened body image and beliefs for girls,[67] earlier and riskier adolescent sexual behavior for boys and girls,[68] and adolescent substance use for boys and girls.[69] Some of the most extensive work has been on the effects of pubertal timing on depression and anxiety, especially the differing effects of early and late pubertal timing.

Theories. Three commonly cited hypotheses attempt to explain the relationship between offset pubertal timing and increased risk of mental health disorders: the maturational disparity, social deviance, and gendered deviation hypotheses.[5] The maturational disparity hypothesis posits that early pubertal timing is detrimental to both boys and girls due to the disparity between their advanced physical development and their relatively lagging cognitive and emotional development.[5] Specifically, the mismatch between advanced development in the limbic system and gradual development in the prefrontal cortex may place early-developing adolescents at a higher risk for mental health disorders.[5, 53, 55] The social deviance hypothesis postulates that offset pubertal timing, be it early or late, leads to an increased risk of mental health disorders.[5] This may be due to the social consequences of being unlike one's peers, such as the stress of feeling different or misunderstood.[55] The gendered deviation hypothesis suggests that early-developing girls and late-developing boys are each at increased risk of mental health disorders.[5] This elevated risk occurs at the intersection of within- and across-gender comparisons, as early maturing girls and late maturing boys constitute the most extreme ends of the entire peer group.[5] While the maturational disparity hypothesis has received much empirical attention and support,[5, 70] late pubertal timing is generally less understood and less studied,[5, 53] and boys are generally underrepresented in puberty research,[71] indicating the need for further research in order to better establish the veracity of these three theories.

Early pubertal timing. Adolescents who experience the onset of puberty earlier than their same-gender and same-age peers have been shown to be at a higher risk for mental health disorders.[5, 9, 70] This relationship has been observed in adolescent samples throughout the world.[5, 72, 73] Both cross-sectional and longitudinal data support the negative correlation between pubertal timing and risk for mental health disorders.[74–75] Furthermore, these effects have been shown to persist into adulthood.[11, 53]

Early pubertal timing has been shown to be an especially significant risk factor for mental health disorders in females, an effect that has been consistently replicated.[5, 53, 55] For example, a 2013 analysis of 3,648 girls from a U.K. birth cohort found a strong relationship between early menarche and increased depressive symptoms in early to mid-adolescence.[76] In general, pubertal timing

research has focused more on female adolescents, and the detrimental effects of early pubertal timing appear to be stronger for females.[5, 55] Because females are two times more likely than males to develop mental health disorders during puberty,[5, 77] the abundance of research regarding mental health in this population may be due to the relative prevalence of those disorders in this gender.

However, some studies have also demonstrated that early pubertal timing is also a strong risk factor for depression and anxiety in boys.[5, 55, 78] A 2017 meta-analysis found that sex did not moderate the small but significant association found between early pubertal timing and mental health disorders, meaning that this relationship did not depend on sex.[5] Some posit that although the magnitude of direct effects of pubertal timing on mental health disorders is similar for boys and girls, there is still reason to believe that the mechanisms that cause this effect may vary by sex.[5] For example, psychosocial mechanisms may include the social and interpersonal skills required to navigate relationships during the pubertal transition. Boys and girls may differ in the extent to which they display such skills, but some researchers suggest that future research is needed to examine possible sex differences.[5]

Four mechanisms have commonly been proposed to explain the relationship between early pubertal timing and increased risk of mental health disorders: hormonal influence, maturation disparity, contextual amplification, and accentuation.[70] First, the hormonal influence hypothesis proposes that the hormonal changes associated with puberty place early-developing adolescents at risk for poor mental health outcomes either due to greater sensitivity to pubertal hormones or because they are exposed to greater amounts of them.[79–80] Second, the maturation disparity hypothesis states that the relationship between early pubertal timing and mental health disorders is a result of the disparity between physical and psychosocial maturity in early-developing adolescents.[70, 80] Third, the contextual amplification hypothesis states that the biological changes associated with puberty interact with stressful environments and amplify environmental stressors, such as peer pressure.[70, 78] Finally, the accentuation hypothesis states that pre-existing emotional and behavioral issues, such as increased negative affect, become amplified due to the stress and uncertainty associated with early pubertal timing.[5, 70, 74, 78]

Late pubertal timing. Compared to the findings on early pubertal timing, research findings related to late pubertal timing are more mixed.[5, 53] Much of the literature seems to agree that late pubertal timing does not appear to make mental health disorders more likely for girls.[5, 81] In fact, some studies have pointed to potential protective effects of late pubertal timing on mental health disorders for girls,[5] and some studies of female adolescents have found a positive correlation between late pubertal timing and factors related to psychological functioning, such as academic achievement.[53, 82]

However, in line with the gendered deviation hypothesis, there is some research that suggests that late pubertal timing has a negative impact on boys.[5, 10, 55, 83–84] For example, a 2009 study found that depression was associated with early pubertal timing in girls and late pubertal timing in boys.[83] Another 2010 study supported the gendered deviation hypothesis in its finding that under high peer stress, both early-developing girls and late-developing boys were at the highest risk for maladjustment.[84] However, in contrast, a 2017 meta-analysis found no significant association between late pubertal timing and mental health disorders for either boys or girls.[5] This disparity is potentially due to the relative lack of research examining late pubertal timing and mental health disorders. Generally, there is a much larger body of literature examining early pubertal timing as opposed to late pubertal timing,[5] and given the lack of research that could be included in the aforementioned meta-analysis, more research is needed on the effects of late pubertal timing to be more confident of its effects.

Measurement. Pubertal timing can be measured using a wide variety of methodologies, which can be split into two categories: one, concerning the timing of milestones of puberty, or two, using self- or parent-reports of pubertal onset.

First, pubertal timing can be measured by determining the onset of certain events associated with pubertal status for an individual. Pubertal status refers to an adolescent's stage of pubertal development, regardless of the onset of puberty.[9, 11] Health providers or researchers typically conduct these assessments of pubertal status.[85] While there is considerable variation in the methodologies used to assess pubertal status,[54] the most commonly used method is the five-stage scales for the development of secondary sexual characteristics throughout puberty known as the Tanner stages.[86–88] Pubertal timing can therefore be indexed by standardizing the measurement of an indicator of pubertal status, e.g. age at menarche, and categorizing the standardized scores of an individual as early, on time, or late. However, there is not necessarily widespread agreement on either what the best milestone to measure pubertal timing is or where the category boundaries should be.[54] Therefore, instead of using categories of onset to measure pubertal timing, some researchers use the residuals from regressing pubertal stage on age in order to conceptualize pubertal timing as a continuous measure in terms of pubertal status, but this method also suffers from the lack of agreement on norms.[54] In particular, pubertal timing in boys suffers from a lack of consensus on methods.[55] Some studies use self-report of age at spermarche, or the onset of sperm emission, as an analogous event to self-report of age at menarche, but researchers disagree concerning whether this is a reliable indicator.[50, 60, 89] Some argue that the most valid indicator of pubertal onset in boys is testicular volume,[54] but this measurement can be perceived as invasive.[55]

Second, subjective measures are also frequently used to measure pubertal timing, such as self-reports and parent reports.[54, 85] These measures are similar to the objective measures discussed above, but they differ in that they are reported by adolescents or parents as opposed to assessed by health providers or researchers.[85] For example, the Pubertal Development Scale allows individuals to disclose the timing of certain pubertal milestones, such as menstruation in females, voice changes in males, and so on.[54, 90] The retrospective report of puberty instrument is similar, but is designed to be reliably used even after the pubertal transition has ended.[91] These methods also suffer from the same lack of consensus on the best milestones to use, particularly for boys.[54]

Summary. Pubertal timing results from the interactions between biological and environmental factors.[9] Offset pubertal timing has been linked to increased risk of developing adolescent mental health disorders.[5] Three generally acknowledged hypotheses offer explanations for this relationship: maturational disparity, social deviance, and gendered deviation[70] Research examining the effects of offset pubertal timing on adolescent mental health disorders has yielded more consistent results for early as opposed to late pubertal timing.[5] Many gaps remain in the literature, including, but not limited to, the effects of late pubertal timing and the most representative and predictive ways of measuring pubertal timing.

Future Directions

The body of research related to puberty has significantly increased in the past decade, particularly in regard to pubertal timing[14, 85] However, much research has tended to focus on the effects of early pubertal timing in adolescent girls.[5] There remains a need to examine the possible effects of late pubertal timing for both boys and girls, to parse pubertal timing and status, and to explore the pubertal experiences of understudied populations.

First, additional research is necessary to further examine the effects of late pubertal timing on adolescent depression and anxiety. Much of the existing pubertal timing research focuses on early as opposed to late pubertal timing, and there are fewer cross-sectional and longitudinal studies on the effects of late pubertal timing on adolescent depression and anxiety.[5] The lack of research related to late pubertal timing may be in part due to the widespread popularity of the aforementioned maturational disparity hypothesis, which only focuses on the detrimental effects of early pubertal timing on adolescent mental health.[5, 70] Widespread acceptance of this idea may discourage research related to late pubertal timing. The lack of research on the effects of late pubertal timing on adolescent depression and anxiety makes it difficult to understand the extent to which this may or may not be a risk factor.

Second, more research is needed on the complex interplay between pubertal timing and pubertal status. Relatively few studies have directly compared pubertal timing and pubertal status.[11] One such study found an association between early pubertal timing and depressive symptoms in girls at age 14, but not at ages 17 or 19,[92] indicating that early-developing girls were more likely to develop depressive symptoms earlier on in the pubertal transition. Because females who experienced early pubertal timing will also be at a more advanced pubertal stage at the age of 14, and research has demonstrated an association between advanced pubertal stage in females and depressive symptoms, status in this example therefore may be the more important contributing factor to these depressive symptoms.[11] While it is difficult to parse pubertal timing and status,[54] such research is needed in order to determine the unique effects of each aspect of puberty on adolescent mental health disorders. Although a large-scale meta-analysis exists on the relationship between pubertal timing and adolescent mental health disorders,[5] no such meta-analysis on the effects of pubertal status exists. Such a comprehensive review may be illuminating, as some research suggests that progression through pubertal stages is a more reliable indicator than pubertal timing of depression in girls.[9, 93]

Finally, further research is needed to evaluate the pubertal experiences of traditionally less studied groups, particularly adolescent boys, low-SES adolescents, racial and ethnic minorities, and LGBTQ+ adolescents.

Adolescent boys: Adolescent boys are greatly underrepresented in puberty research.[71] Some have speculated that the gender disparity in research may be related to the difficulty and uncertainty surrounding the measurement of pubertal timing in boys.[55] There is less consensus on pubertal timing measures for boys than for girls.[54] Furthermore, given that many of the mainstream pubertal timing theories reference gender differences (i.e. the maturational disparity hypothesis and the gendered deviation hypothesis), more research is needed to further examine these hypotheses.

Socioeconomic status: Low-SES adolescents are also understudied in puberty research.[85] Low-SES females often experience puberty earlier than their higher-SES peers.[55, 62–63] Menarche is thought to be related to nutrition and increased body fat,[94] and low-SES girls are at increased risk for obesity due to decreased access to healthy foods.[62, 95] Therefore, more research is needed to better understand the pubertal and mental health experiences of this at-risk population.

Racial and ethnic minorities: In addition, racial and ethnic minorities are also understudied in puberty research.[85, 96] Given that early pubertal timing is more frequently seen in minority groups such as African American and Latino adolescents,[85, 97] more research is needed to better understand the pubertal experiences of racial and ethnic minorities.

LGBTQ+: Lesbian, gay, bisexual, transgender, and queer (LGBTQ+) adolescents are also underrepresented in puberty research.[96, 98] Given that LGBTQ+ adolescents often experience emotional distress as a result of peer discrimination and coming out,[99–100] an increased focus on this population may illuminate the unique experiences of puberty of LGBTQ+ adolescents. In particular, transgender adolescents may have particularly unique experiences with mental health disorders during puberty due to emotional distress resulting from physical development that conflicts with their gender identity.[98, 101]

Conclusion

Adolescent mental health disorders, specifically depression and anxiety, are a prevalent issue in psychology and society. Puberty involves a myriad of complex process, including pubertal timing, which have been shown to be related to increased risk for adolescent depression and anxiety. This review examined the existing evidence for the differential impacts of early and late pubertal timing on adolescent depression and anxiety. This discussion is especially urgent, as pubertal timing is decreasing globally among boys and girls. While stronger evidence exists implicating early pubertal timing in adolescent depression and anxiety, particularly for girls, more research is needed to clarify the mixed findings regarding late pubertal timing. Future pubertal timing research might want to focus on further exploring the potential effects of late pubertal timing on adolescent depression and anxiety, parsing pubertal status and timing, and diversifying samples to better understand the pubertal experiences of understudied groups, such as adolescent boys, low-SES adolescents, racial and ethnic minorities, and LGBTQ+ adolescents.

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A Parametric Study on the Brain Exploring the Role of Hyperelasticity

LUKE HUMPHREY

The brain exhibits both viscoelastic and hyperelastic behaviors (Miller and Chinzei, 2002) [1]. The extent to which the brain exhibits each of these behaviors, however, is not fully known. As more work has been done in this area, a consensus has yet to emerge on material parameters that form a complete, accurate mechanical model of the brain. Models are formed with unique sets of experimental data using various methods, which leads to much variation in the material parameters used across studies. The variation indicates that there is a disagreement on the extent to which certain components of the brain material contribute to the observed behavior. It is likely that the disagreement in parameters will manifest differences between model behavior at extreme loading conditions. Brain behavior at such conditions is pertinent to improving the designs of helmets or crash safety systems. This paper explores the role of hyperelasticity in the brain by comparing the phenomenological differences between a simple linear viscoelastic and hyper-viscoelastic model of the brain. In order to do this, an isotropic model brain was generated using the finite element analysis software *Abaqus* 2019 and rotational loads were applied. A parametric study was performed using this model and the results were analyzed in *Matlab* 9.7. An injury threshold was implemented for each test to reveal differences in material composition. Upon completion of the tests and analysis of the results, a noticeable difference was observed between viscoelastic and hyper-viscoelastic models when comparing resultant shear strains of the tests, particularly at the extreme loading conditions. Noting the observed differences in connection to the material composition will allow researchers to make educated decisions on the extent to which they model brains with hyperelasticity. It will also allow researchers with simple linear viscoelastic models to weigh the potential behavior that might not be shown in the simple models.

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Introduction

Since the 1960's, attaining the most accurate form for modeling the mechanics of the human brain has been studied and the merits of various models have been debated. However, due to the unique characteristics of the human brain and limited ability to conduct tests on it, a consensus has yet to be made in determining its material properties. Because of this, there is much variation in the material parameters used by researchers. Table 1 shows the variation in brain material parameters for Ogden's model of hyperelasticity that have been used in a few well-known publications. The variation in material parameters shown in Table 1 suggests that, under the Ogden model, one set of parameters that models the hyperelasticity of the brain with complete accuracy may not be attainable.

The complexity of the brain has given way to two approaches to researching this topic. The first approach forms models with complexities such as anisotropy and hyper-viscoelasticity, as it is known that these are fundamental aspects of brain behavior (Chatelin et al., 2012) [2]. The second approach models the material composition simply, including only essential components (i.e. density, elasticity, viscoelasticity), so as not to use incorrect parameters for complexities that may dramatically affect results. This approach assumes that if incorrect parameters are used for anisotropy and hyperelasticity, then the results will be less accurate than using a simple model with fewer unknown parameters. Rashid et al., 2012 [3] and many reports like it use the first approach and assume that a hyper-viscoelastic model is necessary in order to produce valid results under certain loads. Other reports have assumed that the added complexities of hyperelasticity or anisotropy are trivial for different circumstances (e.g. Brands et al., 2004) [4]. Either decision can produce meaningful results if the effect of the material assumptions is known and weighed with the conclusions. The following computational studies explores the role of hyperelasticity in the mechanics

α	μ_0 [Pa]	Publication
−4.70	842	Miller and Chinzei, 2002 [1]
6.95	5160	Rashid et. al., 2012 [3]
0.038, 0.063	182, 263	Prange and Margulies, 2002 [9]
3.50, 6.84	319, 137	Valardi et. al., 2005 [10]

Table 1: Variation in Hyperelastic Components. Shown below are the values used for the unrelaxed shear modulus, μ_0, and alpha constant, α, for first order Ogden models of hyperelasticity from four publications. The comma separated values represent parameters for gray and white matter of the brain, respectively.

of the brain under rotational motion so that the effects of material assumptions in brain models are known.

Methods

Two computational studies were conducted using the finite element analysis software package *Abaqus 2019* on a circular cross section of a virtual brain. The first study validated the accuracy of the results from a 0.6 mm mesh model by comparison to the results from a 0.2 mm mesh model. The second study provided a comparison between viscoelastic and hyper-viscoelastic behavior using damage criterion. For both studies, rotational loads were applied to the brain for each material composition and the shear strain response was analyzed.

Geometry. A circular cross section of the brain and skull, modeled with *Abaqus*, was used for all tests and is shown below in Figure 1. The plane strain assumption was used.

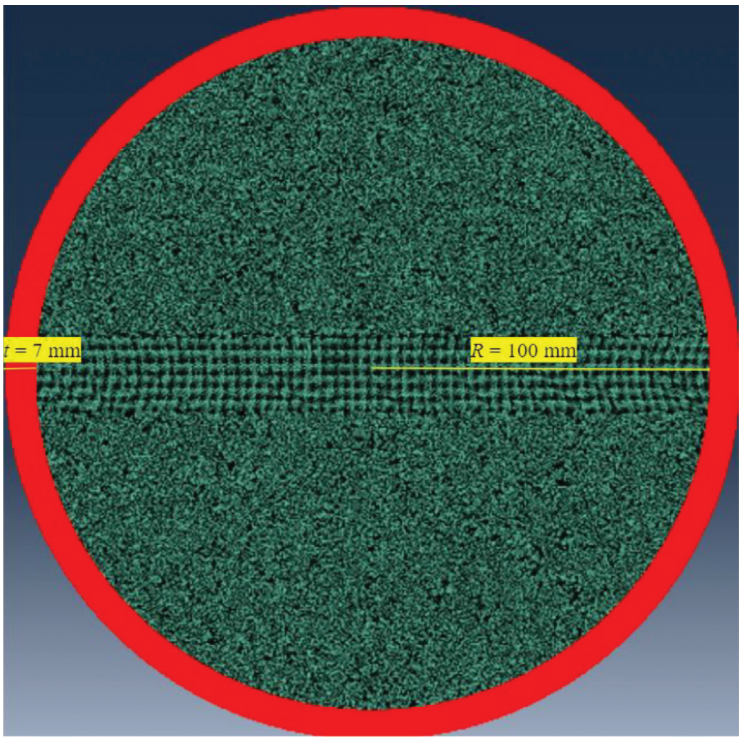
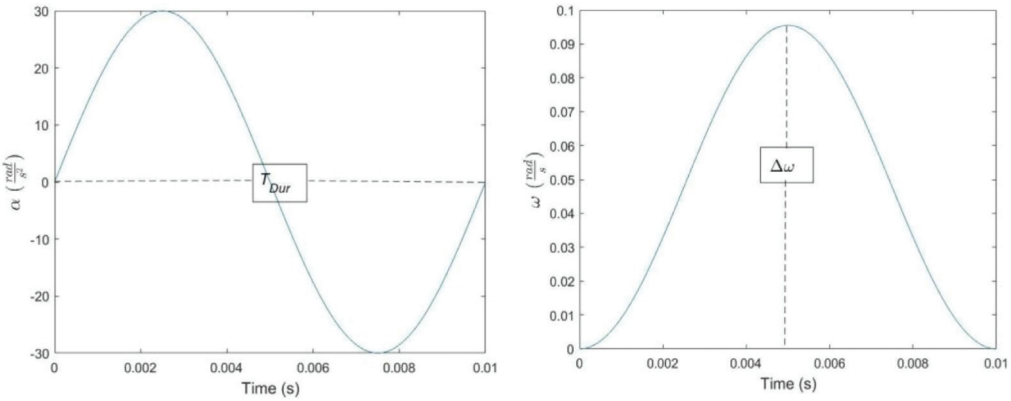


Figure 1: Shown above is the cross section of the *Abaqus* model brain that was used for each test. The skull is shown in red and the brain is in green. The skull is defined as a rigid body and the brain material definition varies between viscoelastic or visco-hyperelastic throughout the studies.

Loading Specifications. For each test, the system was subjected to a load defined by a sinusoidal rotational acceleration about the centroid of the brain. The parameters used to vary the input are T_{Dur} and $\Delta\omega$ as shown below in Figure 2.



(a) (b)

Figure 2: Loading specifications applied to the skull for each test. (a) The rotational acceleration delivered to the skull, with period set by T_{Dur}. (b) The rotational velocity, with magnitude set by $\Delta\omega$.

Ogden's Hyperelastic Model. For the computational studies performed in this report, a first order Ogden model, N=1 was used. The strain energy density equation for Ogden's model of hyperelasticity is shown below in equation (1) [5].

$$U = \sum_{i=1}^{N} \frac{2\mu_i}{\alpha_i^2}\left(\lambda_1^{-\alpha_i} + \lambda_2^{-\alpha_i} + \lambda_3^{-\alpha_i} - 3\right) + \sum_{i=1}^{N} \frac{1}{D_i}\left(J_{el} - 1\right)^{2i} \tag{1}$$

The inputs to this model are the relaxed shear modulus, μ_1, the alpha constant, α, and the D_1 value, which is determined by the bulk modulus, k_0 through the relation $D_1 = \frac{2}{k_0}$. *Abaqus* computes and outputs the total volume change, J_{el}, and stretch values, λ_i, for each finite element of the model.

Mesh Validation

To ensure that appropriate mesh properties were used for the material comparison, four tests with $\Delta\omega$ = 0.10 $\frac{rad}{s}$ and varying values of T_{Dur} = *0.2, 0.3, 0.5, and*

1 ms were conducted for both a 0.6 mm and 0.2 mm brain mesh. The maximum shear strain, γ, was observed at a consistent point in time over a consistent area of the brain for each test. This area is shown in Figure 3 and the material parameters describing the brain are given in Table 2. The viscoelastic component is expressed in the time domain and the hyperelastic component is defined by the Ogden model. The results of the mesh comparison are compiled in Table 3 and reveal the extent to which a 0.6 mm mesh can produce reliable results. A 0.6 mm mesh is preferable to a 0.2 mm mesh as computation can be completed much faster.

Viscoelastic			**Hyperelastic**		
$\underline{g}_i^P$	$\underline{k}_i^P$	τ_i	μ_1	α	D_1
0.84	0	0.1	1000	−5	$2*10^8$

Table 2: Material Composition of Mesh Comparison Model. The material parameters of the model used for the mesh comparison are shown below. The density used for both compositions is 1000 $\frac{kg}{m^3}$.

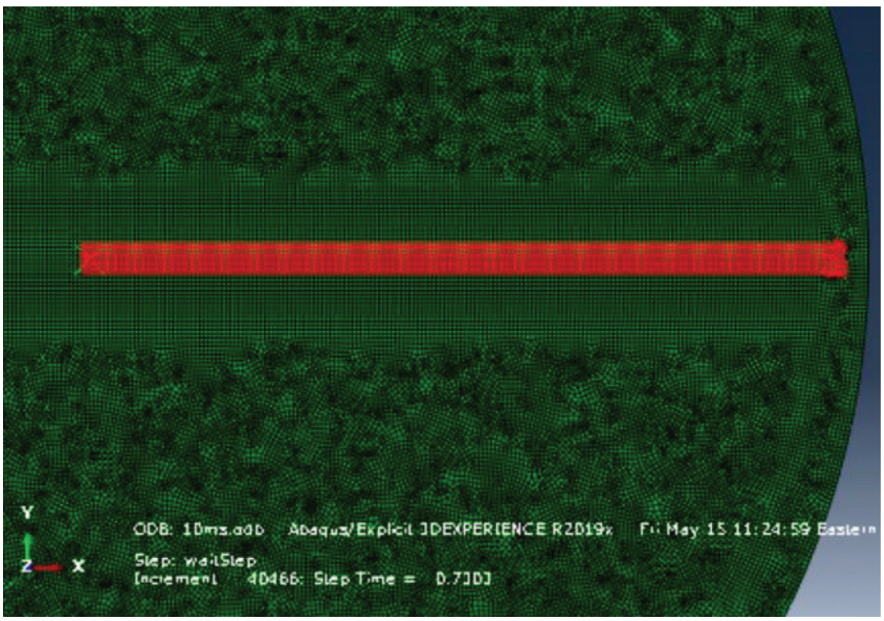


Figure 3: *Shown above in red is the area that was used to compare maximum shear strain values for each mesh in the first study.*

	2 ms Pulse	**3 ms Pulse**	**5 ms Pulse**	**10 ms Pulse**
γ_{max}, 0.2 mm mesh	0.394*10–3	0.587*10–3	0.963*10–3	1.878*10–3
γ_{max}, 0.2 mm mesh	0.225*10–3	0.584*10–3	0.962*10–3	1.874*10–3
Percent Error	**42.9 %**	**0.511 %**	**0.104%**	**0.2130%**

Table 3: Mesh Comparison Findings. Tabulated below are the findings of the mesh comparison. Percent error was calculated under the assumption that the 0.2 mm mesh is accurate. The results from a 0.6 mm mesh become unreliable with pulse durations less than 2 ms.

The findings in Table 3 were used to determine which values of the loading specification, T_{Dur}, produce results that experience insignificant numerical dispersion. Only T_{Dur} values of 3 ms or higher can be relied on to produce accurate results when using a 0.6 mm mesh. Such a condition was used for the remainder of the tests.

Material Comparison Setup

This study compared the behavior of a viscoelastic model with that of a hyper-viscoelastic model. Each test was evaluated against a maximum shear strain injury threshold to reveal differences between the models. A preliminary phase of these tests was completed to show which loading inputs would cause significant deformation of the brain. Once these potentially harmful inputs were known, another phase was completed with more tests using such inputs.

Material Parameters and Loading Inputs. For every set of tests, 9 values of $\Delta\omega$ and 13 values of T_{Dur} were combined, resulting in 117 tests with various loading inputs. Four sets of tests were completed, with each set using a consistent material composition. Lower and upper bounds of the relaxed shear modulus were used in order to understand if this value significantly contributes to differences in shear strain. For each bound of hyper-viscoelasticity, a corresponding composition of viscoelasticity was tested. The values for the corresponding compositions were obtained using Lamé parameter relations as explained below and shown in Table 4. This value was held constant for the lower and upper bounds. The relaxed shear modulus, μ_1, was given a value of 1 kPa for the lower bound and 3 kPa for the upper bound. From these values of μ_1 and K, the corresponding values of the Poisson's ratio, ν, and Young's modulus, E, were calculated using Lamé parameter relations [6].

	Viscoelasticity					Visco-Hyperelasticity					
	$\underline{g}_i^P$	$\underline{k}_i^P$	τ_i	ν	E	$\underline{g}_i^P$	$\underline{k}_i^P$	τ_i	μ_1	α	D_1
Upper Bound	0.84	0	0.1	0.499997	2999.99	0.84	0	0.1	1000	−5	$2*10^8$
Lower Bound	^^	^^	^^	0.4999925	8999.95	^^	^^	^^	3000	^^	^^

Table 4: Material Comparison Parameters. The parameters for each material component are shown below. The viscoelastic component is in the time domain and the hyperelastic component is defined by the Ogden model. The units for each parameter are shown in brackets. The carrots indicate the same value was used for the lower bound as for the upper bound. The density used for all tests is 1000 $\frac{kg}{m^3}$.

The Injury Threshold. In Morrison et. al., 2003 [7], it was suggested that brain cells will experience significant damage if their strain exceeds 10%. For the preliminary phase of tests, the 10% shear strain threshold was used. For the second phase of tests, two shear strain thresholds were used. A lower threshold of was used 5% along with the standard threshold of 10% to reveal whether the effects of the composition differences are exaggerated at higher strains.

Results

Maximum Acceleration Calculations

In order to represent the sets of tests, the maximum angular acceleration, *Max*(α), has been plotted against $\Delta\omega$. The value *Max(a)* for each test is derived from values for $\Delta\omega$ and T_{Dur}. *Shown in Figure 4 is the lower bound of the viscoelastic set of tests from the preliminary phase. On the x-axis of each plot is a logarithmic scale of the maximum angular acceleration, Max*(α), of the skull. On the y-axis is the change in angular velocity, $\Delta\omega$, of the skull. Each point represents a test. Data points shown in red reveal tests where any element on the brain has exceeded the injury threshold.

The results from Figure 4 led to the second phase of tests using loading inputs that were finely incremented within the injury inducing inputs.

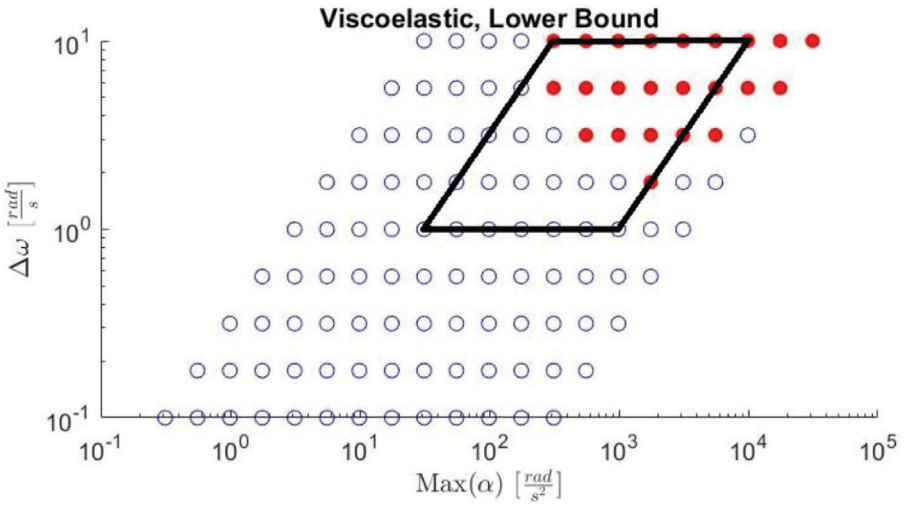


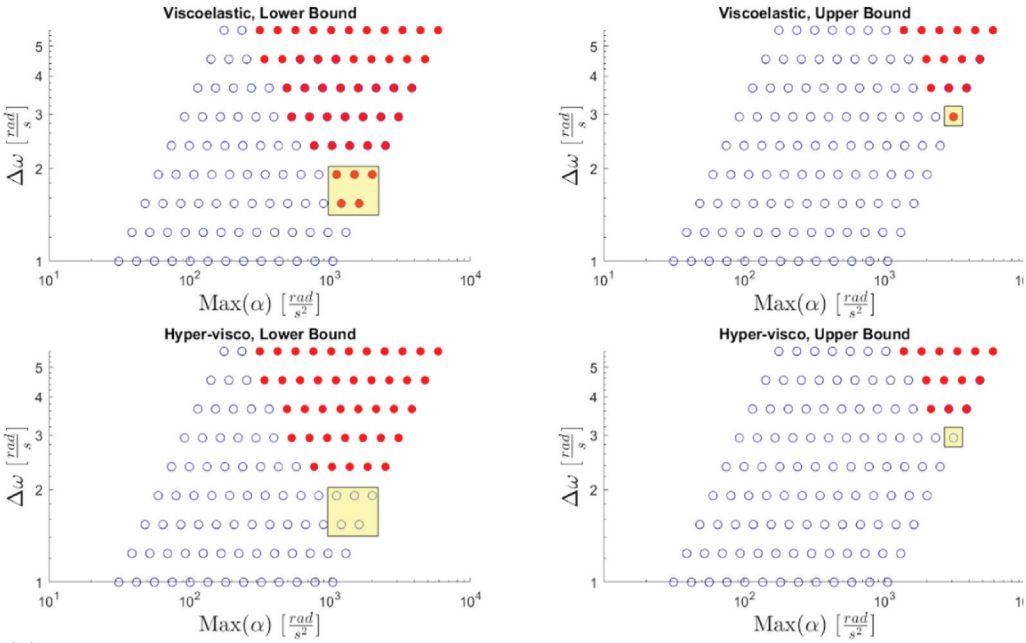
Figure 4: Shown above is the lower bound of the viscoelastic set of tests from the preliminary phase. The black lines show the loading input boundaries for which the second phase of tests are conducted. Notice that the two columns on the right side are excluded due to numerical dispersion, a concept that is explained in the Discussion section.

Material Comparison Results

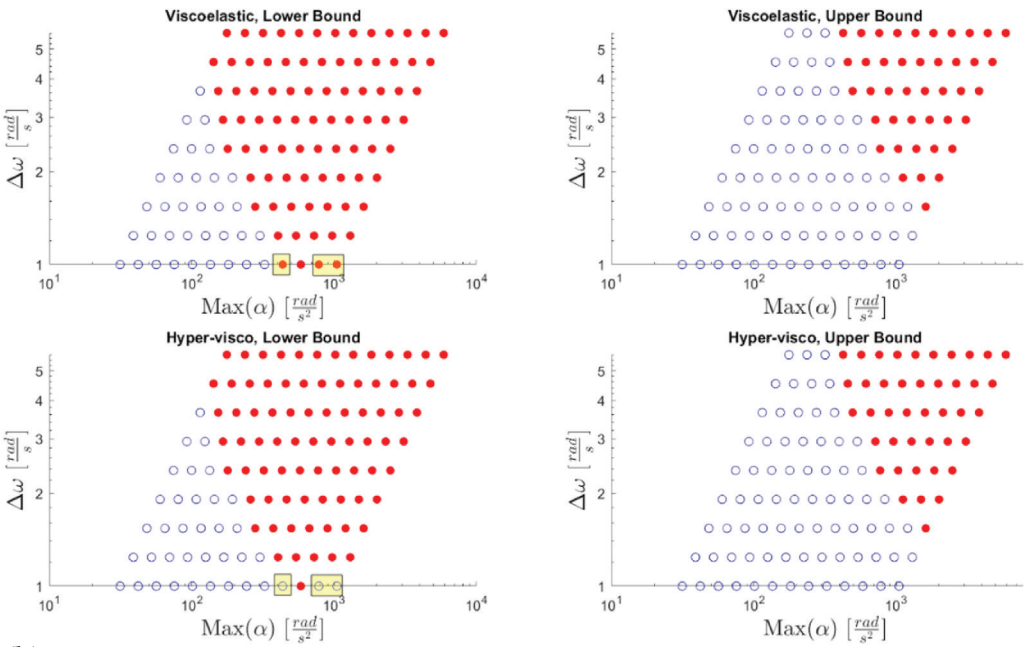
The results of the second phase of tests are compiled in Figure 5 on page 10. The figure uses two different shear strain injury thresholds. Any differences between the visco-hyperelastic and viscoelastic compositions are highlighted in yellow. Three observations can be noted. The first is that a large acceleration ($\alpha > 435 \frac{rad}{s^2}$) is necessary to observe differences between the hyper-viscoelastic and viscoelastic models. Even when the change in angular velocity is high, no differences were observed between the viscoelastic and hyper-viscoelastic models unless significant acceleration was applied. Secondly, the differences caused by the hyperelastic component are more pronounced at higher strains. This is evidenced in that Figure 5a reveals more differences in shear strain behavior than Figure 5b. Lastly, the upper bound, with relaxed shear modulus $\mu_1 = 3$ kPa, has revealed only one difference at an angular acceleration $\alpha > 3{,}000 \frac{rad}{s^2}$. This indicates that the value of the shear modulus significantly contributes to the effect of hyperelasticity.

Discussion

Mesh Consideration. The findings of the mesh comparison, compiled in Table 3, indicate that finite element analysis (FEA) requires a sufficiently fine mesh in



(a)



(b)

Figure 5: Shown above are the results of the tests given an injury condition of (a) 0.1 maximum shear strain and (b) 0.05 max shear strain. Data points in red reveal tests that recorded a maximum shear strain in the brain larger than the threshold. The differences between the viscoelastic and hyper-viscoelastic tests are highlighted in yellow.

order to produce reliable results. This is due to a phenomenon called numerical dispersion. Numerical dispersion occurs in tests when the simulated material exhibits a higher dispersivity than the true material [8]. For the case of modeling the brain, this means that deformation measures, such as strain, which are observed from the model will have a lower magnitude than what would be experienced by a real brain. Because the brain is especially compliant, it will be important for all researchers to check that numerical dispersion does not occur within their FEA models of the brain.

Hyperelastic Effect Dependent on Shear Modulus. Figure 5 on page 10 reveals that the shear modulus plays an important role in determining how hyperelasticity will affect the resultant shear strains. For the upper bound of the relaxed shear modulus, where $\mu_1 = 3$ kPa, only 1 of 16 tests that profile the injury threshold yielded a different result for the two models. This is held in contrast to the lower bound. For this bound, where $\mu_1 = 1$ kPa, 8 of 25 tests that profile the injury threshold yielded different results for the two models. All 9 of the tests that revealed differences between models consistently showed that the hyperelastic model was more resistant to shear deformation than the viscoelastic model.

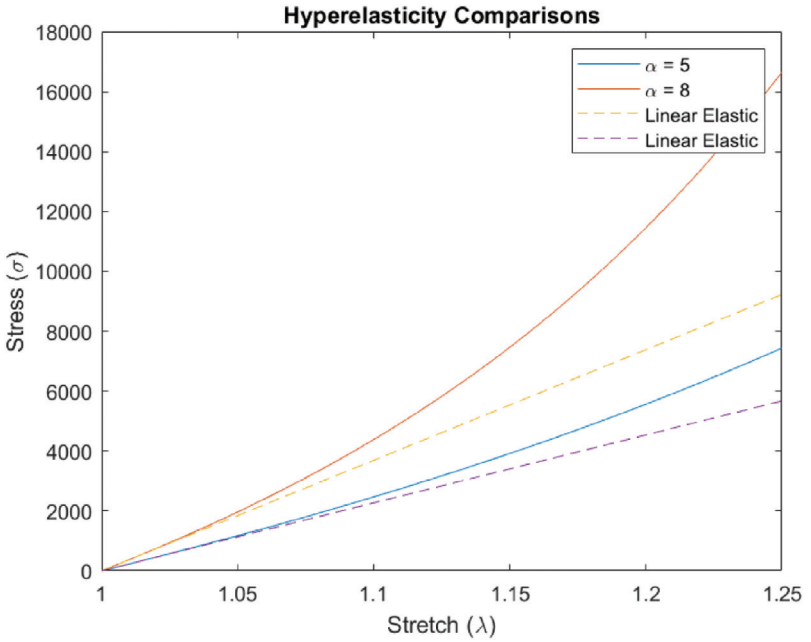


Figure 6: Shown above are two stress-stretch curves of hyperelastic Ogden models in uniaxial tension with corresponding linear elastic stress-stretch curves. As the stretch is increased, the hyperelastic curves demand higher stresses than the linear elastic curves. Also note that as α is increased, this phenomenon is exaggerated.

T_{Dur} [s]	$\Delta\omega$ [$\frac{rad}{s}$]
$10^{-2.5229}$–10^{-1}	10^{0}–$10^{0.75}$

Table 5: Material Comparison Loading Inputs. Evenly spaced values of T_{Dur} and $\Delta\omega$ were selected on a logarithmic scale from the intervals below.

Differences are Revealed at Extreme Loading Inputs. When $\mu_1 = 1$ kPa, the hyperelastic component significantly contributes to the shear strain behavior for loading parameters $\alpha > 450\frac{rad}{s^2}$ and $\Delta\omega < 2.4\frac{rad}{s}$. This shows that quick rotations with high accelerations are where the effects of hyperelasticity are significant.

Hyperelastic Effect Dependent on Shear Strain Threshold. As described in the Methods section on page 7, Figure 5a uses an injury threshold of $\gamma > 0.10$, as this has been considered the shear strain above which concussion is likely. Figure 5b uses a threshold of $\gamma > 0.05$ to study trends of the composition differences with higher strains. Figure 5 shows that the differences become exaggerated between the models as the threshold is increased. This result makes sense when considering the stress-stretch curve of a hyperelastic material. Figure 6 below provides an example of a comparison between a hyperelastic and purely elastic stress-stretch curve, where stretch is defined as $\lambda = \frac{l}{l_0}$. As is shown, the hyperelastic material requires greater stress in order to affect large strains. This attribute of hyperelastic materials helps to explain the resiliency of the models including the hyperelastic component.

Implications for Future Work. If a simple viscoelastic model is being used, then there will be less resistance to large shear strains. This means that if there are observable differences between the model and actuality, it would be that the brain experiences less shear strain than the model predicts. If, on the other hand, a hyper-viscoelastic model is being used, then any differences between the model and actuality would show that the model is more resistant to high shear strains than the true brain. This knowledge lets researchers with simple linear viscoelastic models know that their models tend to overestimate shear strain at extreme loading conditions. Furthermore, hyperelastic models with high shear moduli and large α values are more resistant to shear strain, meaning that the model tends to underpredict shear strain at extreme loading conditions. This is cause for warning as an underprediction in shear strain can lead to flaws in safety mechanism designs that allow for large shear strains to propagate in the brain, which are known to cause injury.

Conclusions

The following conclusions can be drawn from this research.

(1) The hyperelastic tendency is to reduce the maximum strain caused by rotational loads.
(2) The differences between viscoelastic and hyper-viscoelastic models increase as larger shear strains are affected on the brain.
(3) The relaxed shear modulus value contributes to the effect of hyperelasticity on shear strain.
(4) It is at the extreme loading conditions where differences in models become significant. Significant differences between a viscoelastic and hyper-viscoelastic model are observed for tests when the angular acceleration $\alpha > 435 \frac{rad}{s^2}$ and the relaxed shear modulus of the brain is $\mu_1 < 1$ kPa. If a relaxed shear modulus of $\mu_1 > 3$ kPa is being used, the effect of hyperelasticity on shear strain is only observed for loads with angular accelerations $\alpha > 1{,}600 \frac{rad}{s^2}$.
(5) It is crucial for all researchers to ensure that numerical dispersion does not occur within FEA models of the brain.

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UMURJ
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Exploring Quietness in Teams with Bootstrap Analysis

JEONG-HIN CHIN

Introduction

Team-based learning is a type of collaborative learning that is increasingly prevalent throughout all disciplines in higher education (Espey 24; Kim et al. 225–226). It is a common practice for faculty members to include team-based learning into an undergraduate curriculum, and for some, into a postgraduate curriculum. Inclusion of teamwork and team-based assignments can be commonly found in engineering, business, and social sciences programs across the world. Specifically, some faculty members will include team-based assignments in first-year introductory courses as well as final-year capstone projects. Inclusion of teamwork is considered to be beneficial to students in terms of learning to be good team members and this is why Gardner and Korth mentioned that "To remain innovative and competitive, businesses are looking for employees who can work and learn effectively in teams" (28). Besides, previous studies have shown that by learning in teams, students' academic achievement and self-efficacy may increase.

Nonetheless, team-based learning is not a universally positive experience for all students, as some of the obstacles in teamwork include communication difficulties, uneven work allocation, free-riders and unfair grading experiences. (Wilson et al. 794; Pfaff and Huddleston 38; Medaille and Usinger 240–42). As teams are often made up of students who come from different backgrounds, it is normal for them to worry about potential obstacles in team-based learning. For example, all ten participants interviewed as part of Medaille and Usinger's study noted that they had negative experiences with team-based collaborative learning due to the presence of free riders in their groups (Medaille and Usinger 246). In another study, non-high-achieving students were found to have difficulties in expressing their ideas, as high-achieving students in the groups had prepared

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for the projects on their own and were more likely to persuade the other students to follow their ideas rather than negotiating to resolve any conflicts. (Lee et al. 423; Lee and Lim 222)

In order to combat the perceived inequalities and negative experiences expressed by students regarding team-based projects, faculty members have introduced and developed techniques to assist in the teamwork process. (Chin et al. 3). Both the use of computer-supported collaborative learning environments and the use of peer mentors to assist teams throughout the semester have shown beneficial outcomes for students on teams. (Chin et al. 4; Ruël et al. 17–18). Teamwork assessment and support tools such as CATME encourage students to rate their teammates and themselves, while instructors can easily retrieve large amounts of data gathered by the tools. (Beigpourian et al. 11; Chin et al. 5; Maneeratana and Sripakagorn 5). With the help of these tools, instructors are able to look at one of the main aspects of a student's negative experience of students in team-based learning, such as communication difficulties, and try to help the team out if there are disruptions in team dynamics (Beigpourian et al. 11)

Communication difficulties in teams can be due to students being reluctant to share their thoughts or just being shy and introverted. These may be the reasons why students are quiet in teams, but other reasons for being silent are often the result of personal, social, academic, cultural and contextual constraints (Medaille and Usinger). While previous studies have explored quiet students' behaviors (Jin) and how they perceive themselves in collaborative learning (Medaille and Usinger), this study was designed to understand the following research questions (RQ):

RQ 1: To investigate if there is a relationship between three variables measuring various communication-related metrics on a beginning-of-term survey: "Extraversion" (self-rating of a student speaking up in groups), "BT_BelongingConcern" (students' beginning of term concern regarding fit in the course), & the variable "SpeakUp" (self-rating of how likely they will hold back ideas to ensure other group members stay happy).

RQ 2: To investigate whether there is a relationship between self-rated previous team experiences (number and positive/negative valence) for the three variables mentioned. It is crucial for instructors to understand these two questions so that they can divide the students into groups that fit the students' personality and traits based on the students' responses to a survey administered at the beginning of term, rather than randomly grouping students. In order to test the research questions mentioned above, I propose the following hypothesis to be investigated in this study:

Hypothesis 1: There are significant correlations between students' self-rated likelihood of speaking up in groups and (Extraversion), students' beginning of term belonging concern score (BT_BelongingConcern), as well as between students' self-rated likelihood of speaking up in groups and students' self-rated likelihood of holding back ideas to ensure other group members stay happy (SpeakUp).

It is common to have students who tend to listen more and speak less in a group. These students are considered quiet and will often express agreement with the thoughts of others regardless of whether they actually agree with the ideas (Medaille and Usinger 242; *Avoiding Communication* 149–53). Moreover, students who talk a lot are seen to "dominate the interaction of the group" and "be quite willing to disagree with other group members" (*Avoiding Communication* 149–53). Medaille and Usinger also mentioned that quiet students suffer from "tensions between speaking and silence, engaging and disengaging and belonging and isolation when interacting with group members." (254). I believe that a student's initial perception towards a course before the term has begun will have an effect on how the students perceive their extraversion score in the course. Thus, the null and alternate hypotheses are:

$$H_{0_1} : r = 0$$

$$H_{1_1} : r \neq 0$$

While previous studies find these relationships (Medaille and Usinger 254), it is important to reproduce this finding quantitatively. Thus, by calculating the correlation between variables to find the R-squared value, we are able to be more confident in saying that there is a relationship between "Extraversion" & "BT_BelongingConcern" and "Extraversion" & "SpeakUp" among the students who responded to the survey.

I will be using Kendall's tau-b (τb) statistic to calculate the correlation between the variables mentioned above. τb is chosen over other statistics to calculate the correlation as τb is the better in calculating the correlation between two ordinal variables (Khamis 159) as the questions in the survey were in the form of seven-point scales with identified end points.

Hypothesis 2a: A student with many previous teamwork experiences is more likely to score high in "Extraversion", high in "SpeakUp," and low in "BT_BelongingConcern."

Hypothesis 2b: A student with past positive teamwork experiences is more likely to score high in "Extraversion", high in "SpeakUp," and low in "BT_BelongingConcern."

For the two hypotheses above, I believe that when a student has many teamwork experiences or has had prior good experience with teamwork, they are able to recognize the benefit obtained through collaborative learning in groups such as increased individual achievement and persistence when facing adversity (Pfaff and Huddleston 38). Many prior teamwork experiences might have given students the confidence and more understanding of what to expect from a class that contains team projects while prior positive experiences might make the students want to work as a team again. Thus, both of the 2a and 2b hypotheses will use the same null and alternative hypotheses but will be tested using different smaller filtered data sets (as will be explained in the Methods section) to fit the condition of Much Experience vs. Less Experience and Positive Experience vs. Negative Experience.

$$H_{0_{3a}}: \Delta_{Extraversion} = 0,\ H_{0_{3b}}: \Delta_{Belongingness} = 0, \text{ and } H_{0_{3c}}: \Delta_{Speakup} = 0$$

$$H_{1_{3a}}: \Delta_{Extraversion} \neq 0,\ H_{1_{3b}}: \Delta_{Belongingness} \neq 0, \text{ and } H_{1_{3c}}: \Delta_{Speakup} \neq 0$$

After testing the three hypotheses mentioned above, I will continue this paper by calculating which predictors have a lower loss in predicting the variable "Extraversion" (rate of a student speaking up in groups) and will also perform cluster analysis on the students to facilitate team formation in the future.

Data

Data Collection

The data for this study was collected from 2088 students enrolling in Engineering, Business, Informatics, and Architecture courses at the University of Michigan using a team assessment tool. The students answered a Beginning of Term survey before they were put into groups by their respective instructors. Therefore, the survey used in this research will collect the students' personality and traits before the semester started and before they were divided up into teams. Although the students are from different courses, a similarity between these students is that the courses are conducted in a team-based collaborative learning format. In each course, students are required to complete project(s) assigned by the instructors in their respective teams. Projects vary by courses, but most of the projects require students to brainstorm ideas, solve challenges, and present their findings or products.

Data Cleaning

The survey forms contained different types of questions that vary from courses to courses. Among the questions included in the initial survey, 13 of them are the same across the 17 different courses. The final cleaned data is stored in a single file containing only the responses to the 13 questions. Note that while the team assessment tool included more questions and other assignments that students had to complete each week, only five variables from the responses are studied for the purposes of this project. In the original data file, six participants that contain NA values in one or more variables were removed in order to prevent error from occurring, leaving us with 2082 responses. Since the total number of samples collected was 2088, the 6 samples removed will not affect the computation in any important way. Some basic analysis on the five variables can be seen in Table 1, Table 2 and Figure 1.

Figure 3 to 5 tells us about the distribution of students' responses in the three different variables. Nonetheless, these plots are not interesting by themselves as they do not tell us about the interaction between the variables. Therefore, to get a clearer picture of how the other two variables interact with Extraversion, two density plots were plotted to look at the distribution of students in "Extraversion v.s. SpeakUp" and "Extraversion v.s. BT_BelongingConcern".

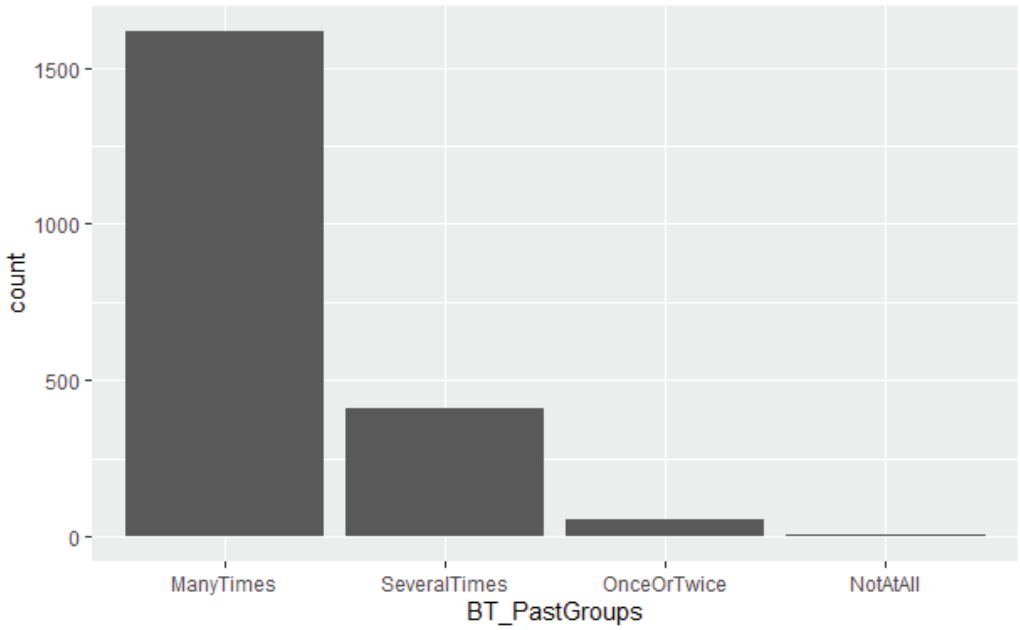

Figure 1: Most students report many team experiences.

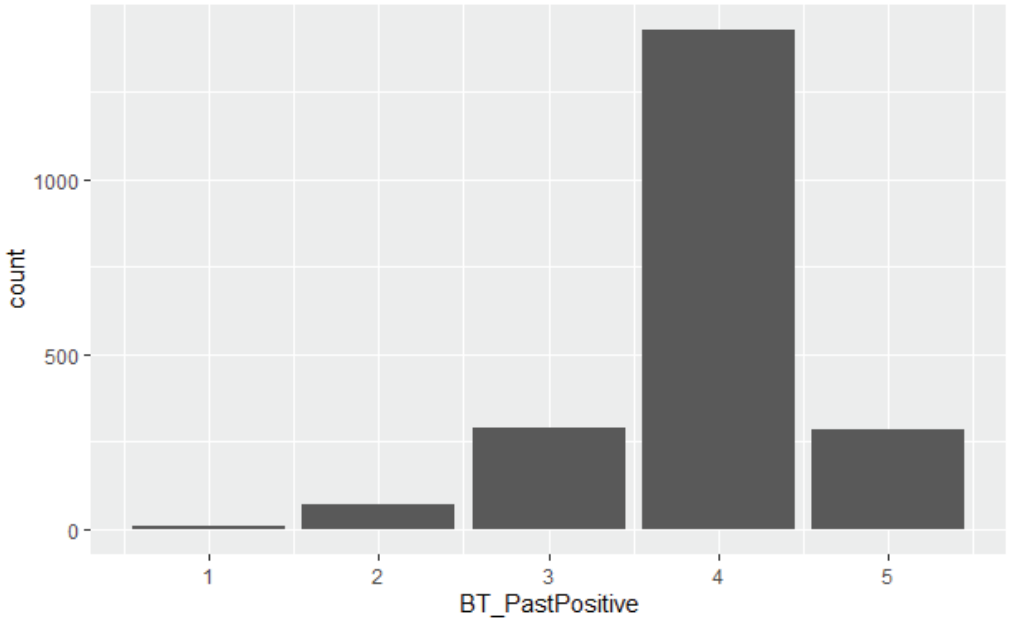

Figure 2: Most students report fairly positive (4) or very positive (5) experiences in their previous teamwork.

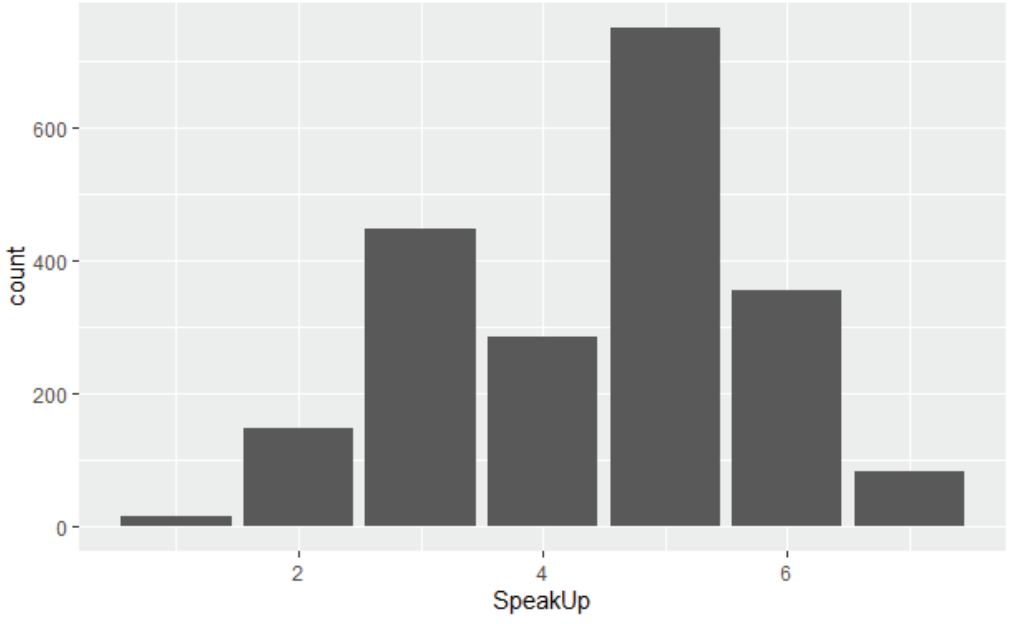

Figure 3: The SpeakUp variable appears bimodally distributed, with more people toward the "It's easy for me to speak up about my ideas or preferences even if it disrupts my group" end of the scale than the "I'd rather hold back ideas or preferences if my group stays happy" end.

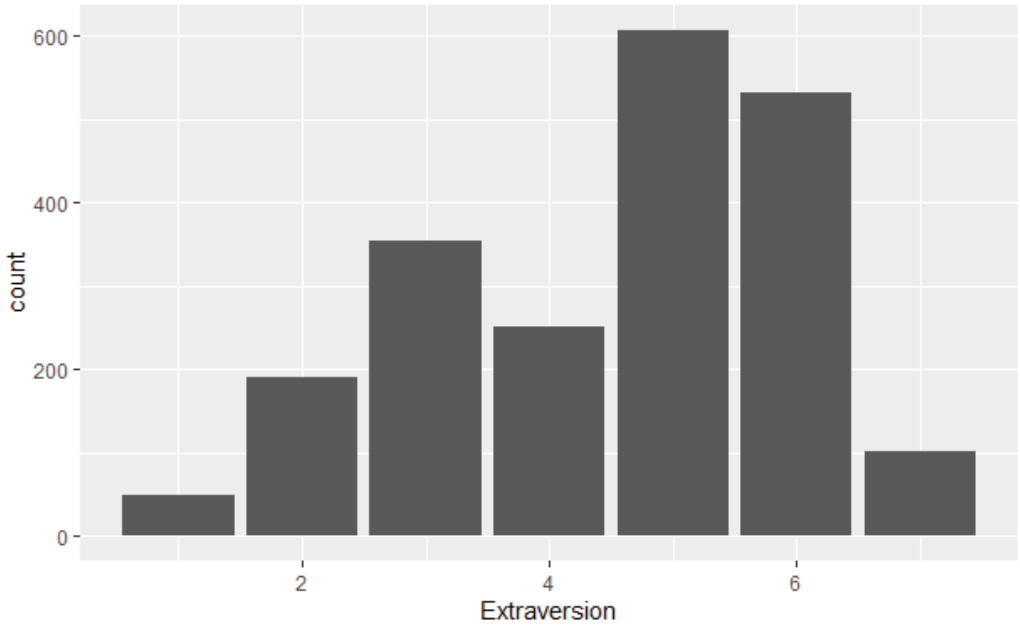

Figure 4: The Extraversion variable appears bimodally distributed, with more people toward the "I often speak up in groups" end of the scale than the "I tend to listen more than speak" end.

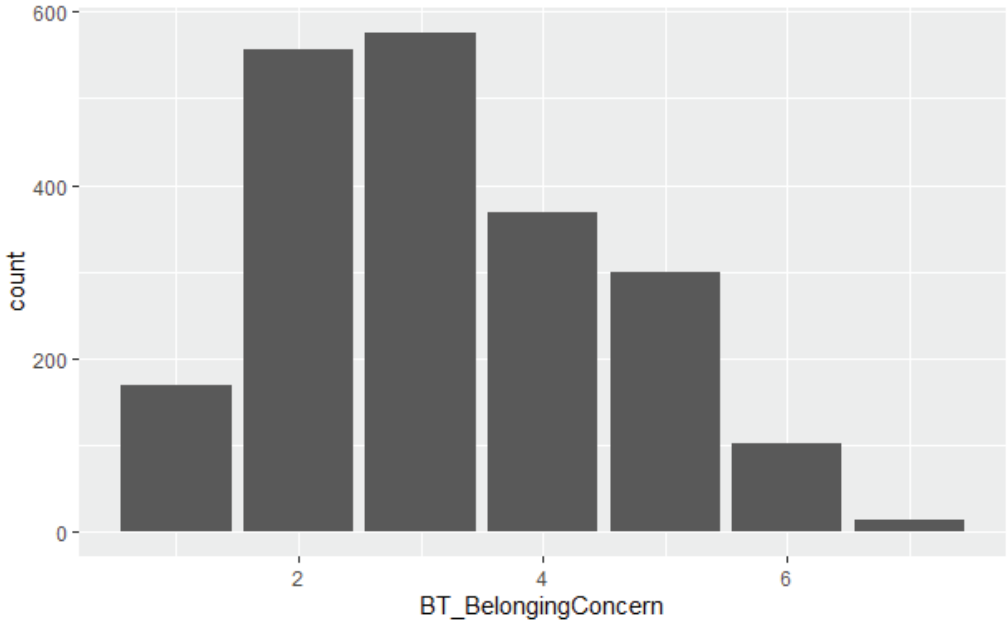

Figure 5: The BT_BelongingConcern variable appears to be right-skewed, with more people toward the "I expect to fit right into the course" end of the scale than the "I expect to feel pretty out of place in the course".

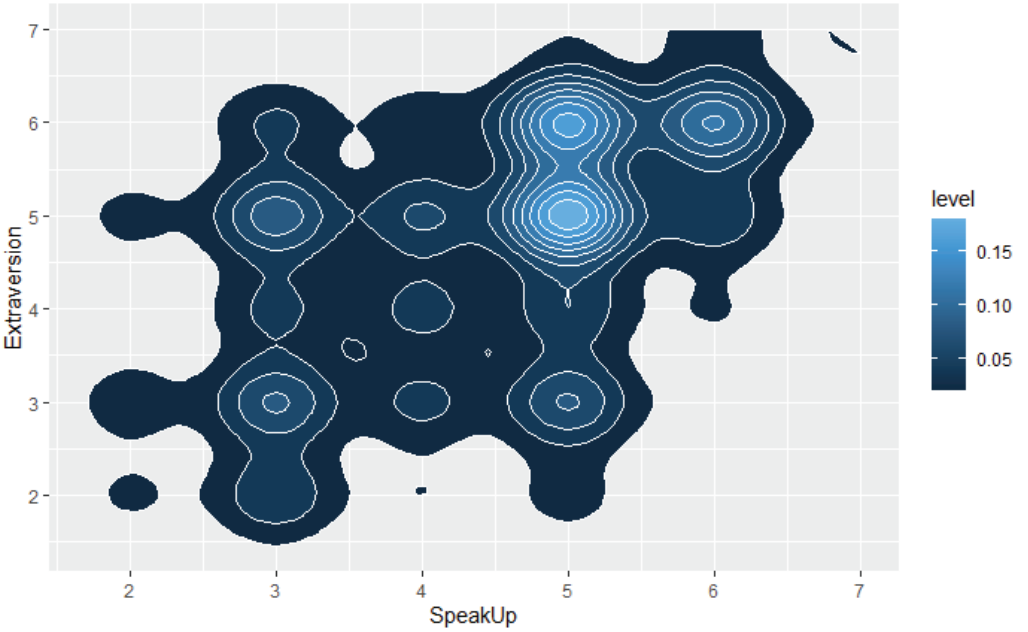

Figure 6: The top-right portion of the density plot is denser, with more students toward the "It's easy for me to speak up about my ideas or preferences even if it disrupts my group" and "I often speak up in groups" end.

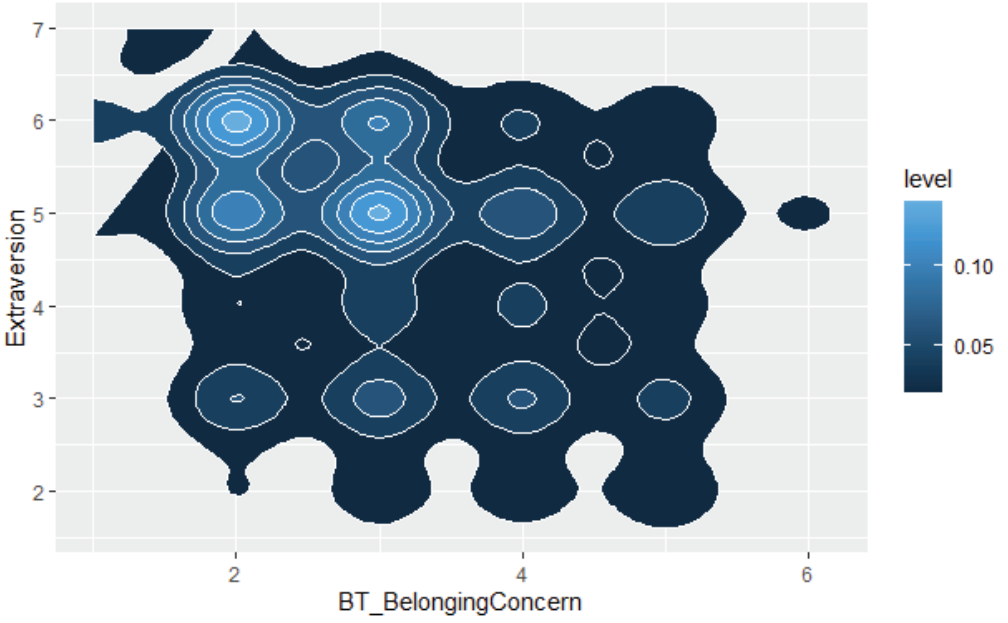

Figure 7: The top-left portion of the density plot is denser, with more students toward the "I expect to fit right into the course" and "I often speak up in groups" end.

Methods

Instrument Development

For this study, I used the "Beginning of Team" (BoT) survey which comprised a total of N = 2082 students (6 students' responses were removed as mentioned above). The students responded to the following survey questions at the beginning of the Fall 2020 semester. Ordinal data was converted into numerical data so that computation can be carried out easily. I analyzed all the following variables in the range of 1–7 (or 5 in some cases) without performing any modification such as mid-ranking as I believed the students had clearly expressed their opinion using the Likert scales. The variables that were studied in this paper and its details are as:

[Extraversion] Students rated themselves on a 7-point scale to this statement. 1-point for this statement translates to "In groups, I tend to listen more than speak" and a 7-point translates to "I often speak up in groups".

[SpeakUp] Students rated themselves on a Likert 7-point scale to this statement. 1-point for this statement translates to "I'd rather hold back ideas or preferences if my group stays happy" and a 7-point translates to "It's easy for me to speak up about my ideas or preferences even if it disrupts my group".

[BT_BelongingConcern] Students rated themselves on a Likert 7-point scale to this statement. 1-point for this statement translates to "I expect to fit right into the course" and a 7-point translates to "I expect to feel pretty out of place in the course". Note that BT stands for Before Term, so this metric is about how the students feel about themselves fitting into the course before the class has actually begun.

[BT_PastGroups] Students answered this statement which asks about whether they have past experiences with teamwork. The students answered this question with either "Not at all", "Once or Twice", "Several Times", or "Many Times".

[BT_PastPositive] Students rated themselves on a Likert 5-point scale to this statement, ranging from "Strongly Disagree" to "Strongly Agree," to the statement, "My previous teamwork experiences were generally positive." A student's past teamwork experience is considered to be positive if he or she scores this item as "agree" or "strongly agree".

This paper aims to study students' self-rated "Extraversion" (tendency of students being quiet or speaking up in a group) and to understand what might affect students' perception that they will be quiet in a team. Thus, "SpeakUp" and "BT_BelongingConcern" are chosen as predictors since they have the highest correlation coefficient with "Extraversion" as observed in Table 1. "BT_PastGroups" and "BT_PastPositive" are used to separate the data sets into smaller sets, where the separation method is explained above.

Analysis

To answer the research questions, I first performed data visualization to see if there are any interesting trends among the data. As the data are discrete, most of the points ended up overlapping one another, thus a density plot in the form of a heat map was chosen so that the trends can be observed clearly as in Figure 6 and 7. In these plots, all 2082 students' responses were used as I wanted to see how students rated themselves in terms of "Extraversion", "BT_BelongingConcern", and "SpeakUp". To test the first hypotheses, the correlations, τb, between "Extraversion" and "BT_BelongingConcern" and "Extraversion" and "SpeakUp" are calculated. Since this study focuses on the analysis of ordinal data, I will be using the Stuart-Kendall Tau-b to calculate the correlation coefficient rather than Pearson's *r* (Khamis 159). The correlation is calculated using R's *KendallTauB* function.

After looking at the mean score of "SpeakUp", "BT_BelongingConcern", and "Extraversion", I was curious to find the population mean for these variables. In order to approximate the means, I used the Bootstrap method. The 95% confidence interval was calculated using R's in-built boot.ci() function. As the sample size was large and the three variables were approximately normally distributed, I used the normal confidence interval of the function. The normal confidence interval can be expressed as the following:

$$T \pm Z_{\frac{\alpha}{2}} \sigma_{T^*} \tag{1}$$

$$\hat{\theta} = T = T\left(X_1, X_2, \ldots X_n\right),$$

$T^* = T\left(X_1^*, X_2^*, \ldots, X_n^*\right)$ given that $X_1^*, X_2^*, \ldots, X_n^*$ are the Bootstrap samples

σ_{T^*} is the standard error for the Bootstrap samples

$Z_{\frac{\alpha}{2}}$ is the 100 $(\frac{\alpha}{2})^{\text{th}}$ and 100 $(1\text{-}\frac{\alpha}{2})^{\text{th}}$ quantile of standard normal distribution

After obtaining the basic information about the three variables to be tested, bootstrap is once again used to test the third hypothesis by bootstrapping 10000 times (B=10000). The data was separated out into four smaller datasets, which were data of students with many teamwork experiences (6 or more times) and students with less or no teamwork experience (less than 6 times); and data of students with past negative experiences (score of 3 or less) and students with past positive teamwork experience (score of 4 or more). For hypothesis 2a, the difference in means for the "SpeakUp", "BT_BelongingConcern", and "Extraversion" variables were calculated through:

$$\Delta_{Speakup} = E\left(M.Teamwork\left[SpeakUp\right]\right) - E\left(L.\ Teamwork\left[SpeakUp\right]\right)$$

$$\Delta_{Belongingness} = E\left(M.Teamwork\left[Belongingness\right]\right) - E\left(L.Teamwork\left[Belongingness\right]\right)$$

$$\Delta_{Extraversion} = E\left(M.Teamwork\left[Extraversion\right]\right) - E\left(L.\ Teamwork\left[Extraversion\right]\right)$$

where M means many while L means less.

For hypothesis 2b, the difference in means for the "SpeakUp", "BT_Belongingness", and "Extraversion" variables are calculated through:

$$\Delta_{SpeakUp} = E\left(P.Experiences\left[SpeakUp\right]\right) - E\left(N.Experiences\left[SpeakUp\right]\right)$$

$$\Delta_{Belongingness} = E\left(P.Experiences\left[Belongingness\right]\right) - E\left(N.Experiences\left[Belongingness\right]\right)$$

$$\Delta_{Extraversion} = E\left(P.Experiences\left[Extraversion\right]\right) - E\left(N.\ Experiences\left[Extraversion\right]\right)$$

where P means positive while N means negative. For each of the differences in means, their respective confidence intervals are calculated using the same normal boot.ci() function.

Moving on, assuming that for some reasons, the instructors were unable to access the students' Extraversion scores and had to predict students' rate of speaking in groups based on other variables collected in the survey form, then it is essential to figure out what variable is the best predictor in determining a student's Extraversion score. In order to achieve this, Leave-One-Out Cross-Validation can be used. In this study, a cost of one would be paid if quiet students are classified as talkative and vice-versa. The cross validation implementation uses 10000 replications to determine which of the two variables has the lowest average loss.

In addition, hierarchical clustering is used to cluster the students so that when new points (future students' responses) are obtained, instructors will have a better understanding of which clusters the students belong to, easing lecturers in the process of assigning the students into teams.

Result And Analysis

Before I start testing the hypotheses, performing prediction or clustering the data, I first computed the correlation between each variable and the "Extraversion" variable to determine the best two predictors to be used. Thus, according to Table 1, we see that the top two predictors are "SpeakUp" and "BT_Belongingness". Do note that the correlation coefficient value in Table 5 is the absolute value of the original value.

| cor.val |	**Predictors**
1	Extraversion
0.3485	SpeakUp
0.2217	BT_Belongingness
0.168	ManyTeamEXP
0.1056	BT_PastPositive
0.0747	Procrastination
0.0654	Group_Preference
0.0481	PositiveExp
0.0447	Control
0.0337	BT_PastDiverse
0.0033	BT_PastWorkDifferent

Table 1: Correlation coefficient between variables and "Extraversion"

After finding the variables that have the highest correlations with "Extraversion", the sample mean, bootstrapped mean confidence interval, bias and mean squared error (MSE) of "Extraversion", "SpeakUp", and "BT_Belongingness" are calculated as shown in Table 6.

Variable	Mean	Bootstrap C.I. (95%)	Bias	MSE
Extraversion	4.5269	(4.463, 4.591)	–0.00013	0.001073
SpeakUp	4.4424	(4.386, 4.499)	–0.0001845	0.0008213
BT_BelongingConcern	3.2051	(3.147, 3.263)	0.0001997	0.00087

Table 2: Mean, confidence interval, bias, and MSE of the "Extraversion", "SpeakUp", and "BT_BelongingConcern" for overall sample

Hypothesis 1: There is a correlation between the metric about the rate of students speaking up in groups (Extraversion) and the metric about students' beginning of term belonging concern score (BT_BelongingConcern) and the metric about whether the students will hold back ideas to ensure other group members stay happy (SpeakUp).

From Table 1, it is noticeable that the correlation between "Extraversion" and "SpeakUp" is not zero. Using the Stuart-Kendall Tau-b, the correlation between "Extraversion" and "SpeakUp" is around $\tau_b = 0.32$ with a confidence interval of (0.3160, 0.3810). Therefore, since 0 is not included in the confidence interval, I will reject the null hypotheses, H_{0_1} in favor of the alternate hypotheses. This result actually agrees with the conclusions obtained in previous research where quiet students will often express agreement while talkative students are seen to be quite willing to disagree with other group members.

From Table 1, it is noticeable that the correlation between "Extraversion" and "BT_BelongingConcern" is not zero and by using the Stuart-Kendall Tau-b, the correlation between "Extraversion" and "BT_BelongingConcern" is around $\tau_b = -0.2217$ with a confidence interval of (–0.2560, –0.1875). Therefore, since 0 is not included in the confidence interval, I will reject the null hypotheses, H_{0_2}, in favor of the alternate hypotheses. This result also agrees with the conclusion from previous research that quiet students suffer from tensions between belonging and isolation when interacting with group members (Medaille and Usinger 254).

Variable	Δ	Bootstrapped C.I. (95%)
Extraversion	1.989247	(0.976, 6.086)
SpeakUp	0.8129032	(–1.2603, 3.8474)
BT_BelongingConcern	–1.651613	(–5.334, –0.147)

Table 3: Difference in mean between students with many teamwork experiences and less teamwork experience and confidence interval

Hypothesis 2a: A student with many previous teamwork experiences is more likely to score high in "Extraversion", high in "SpeakUp" and low in "BT_BelongingConcern"

From Table 3, it is observable that there is a difference in mean in the "Extraversion" and "BT_BelongingConcern" variable between students with many teamwork experiences and less teamwork experience. This is because the bootstrapped confidence intervals for these two variables do not include 0 in them. Students with many teamwork experiences score an average of almost 2 points higher in terms of extraversion, meaning that they self-rate as more often speaking in groups. Moreover, students with many teamwork experiences score almost 1.7 points less in terms of BT_BelongingConcern, meaning that they are more likely to expect to fit right into the course when the course has just started. I believe this is due to previous teamwork experiences giving students the confidence to express themselves in groups and reduce their fear towards teamwork projects in the new course. Although the difference in mean for the "SpeakUp" variable is about 0.8, I am unable to conclude that there's actually a difference in "SpeakUp" variable between the two types of students due to the fact that the confidence interval contains 0. I believe that other factors, such as respecting others' opinion or cultural constraints (Medaille and Usinger 243) might be more important than teamwork experiences.

Thus, I will reject the null hypothesis in favor of the alternate hypothesis that there is indeed a difference in the mean score for "Extraversion" and "BT_BelongingConcern" among the two types of students. However, there is no clear evidence for me to reject the null hypothesis to conclude that there is a difference in mean score for the "SpeakUp" variable among the two types of students.

Variable	**Δ**	**Bootstrapped C.I. (95%)**
Extraversion	1.674797	(0.218, 5.976)
SpeakUp	0.6747967	(-1.3437, 3.7271)
BT_BelongingConcern	-1.658537	(-5.416, -0.157)

Table 4: Difference in mean between students with positive teamwork experiences and negative teamwork experience and confidence interval

Hypothesis 2b: A student with past positive teamwork experiences is more likely to score high in "Extraversion", high in "SpeakUp" and low in "BT_BelongingConcern"

From Table 8, it is observable that there is a difference in mean in the "Extraversion" and "BT_BelongingConcern" variables between students with positive

teamwork experiences and negative teamwork experiences. Surprisingly, the result from this test is similar to the ones obtained from Hypothesis 2a. The bootstrapped confidence intervals for "Extraversion" and "BT_Belongingness" do not include 0 in them. Students with past positive experiences score 1.7 points higher in terms of extraversion, meaning that they identify as often speaking in groups, and 1.7 points lesser in terms of BT_BelongingConcern, meaning that they expect to fit right into the course when the course has just started. I believe that past positive teamwork experiences have a similar effect as many teamwork experiences in terms of giving students the confidence to express themselves in groups and reduce their fear towards teamwork projects in the new course. Once again, I believe that Medaille and Usinger's explanation can be applied here to explain why there is no clear mean difference in the "SpeakUp" variable between the two types of students (243). Thus, I will reject the null hypothesis in favor of the alternate hypothesis that there is indeed a difference in the mean score for "Extraversion" and "BT_BelongingConcern" among the two types of students. However, since the confidence interval does include 0 for the "SpeakUp" variable, there is no clear evidence for me to reject the null hypothesis that there is a difference in mean score for the "SpeakUp" variable among the two types of students.

Variable	**Average Loss**	**Average Cut-off point**
SpeakUp	0.4544	6.0303
BT_BelongingConcern	0.3823	3.0606

Table 5: Average Loss and cut-off points for "SpeakUp" and "BT_BelongingConcern" variable

As mentioned in the introduction, assuming that the lecturers are unable to access the "Extraversion" score directly due to some reasons, then a variable with the least average loss must be chosen as the predictor to predict whether a student is expected to be quiet (Extraversion score is 4 or below) or talkative (Extraversion score is 5 or above). By computing the average loss for both of the variables with the highest correlation coefficient with "Extraversion", it is observable that "BT_BelongingConcern" has a lesser average loss in predicting whether a student is quiet or not, even though the semantic relatedness with "SpeakUp" is high. Moreover, since the average cut-off point is 3.06, it means that students scoring a point of 4 or more in BT_BelongingConcern have a (1–0.3823)*100% = 61.77% likelihood of being a quiet student since "BT_Belongingness" and "Extraversion" are negatively correlated. However, if "SpeakUp" is used as a predictor, then only a student who scores a point of 7 out of 7 has a (1–0.4543708)*100% = 54.5629% accuracy of being a talkative student. Therefore,

the result shows that "BT_BelongingConcern" is a better predictor in predicting whether a student is quiet or not. I believe that this statement makes sense as if a student feels that he or she does not fit into a class, then the student might choose to be quiet in it. (Medaille and Usinger 254)

Lastly, hierarchical clustering is used to plot the distribution of students' responses. Figure 5 shows the dendrogram for the hierarchical clustering. From the dendrogram, I chose to separate the data out into two smaller clusters as shown in Figure 6. According to Table 10, it is noticeable that if new data points fall into the second cluster (on the left in Figure 6), the probability of students being quiet increase (69% of the new points will be quiet); if the points are in the first cluster (on the right in Figure 6), the probability of students being quiet decreases, (16% of the new points will be quiet).

Cluster	Model coefficients	Probability
1	(Intercept) xTRUE 0.8079 -2.4311	0.1648
2	(Intercept) xTRUE -1.6232 2.4311	0.6917

Table 6: Model coefficients of each cluster and the probability of students being quiet if categorized into each cluster

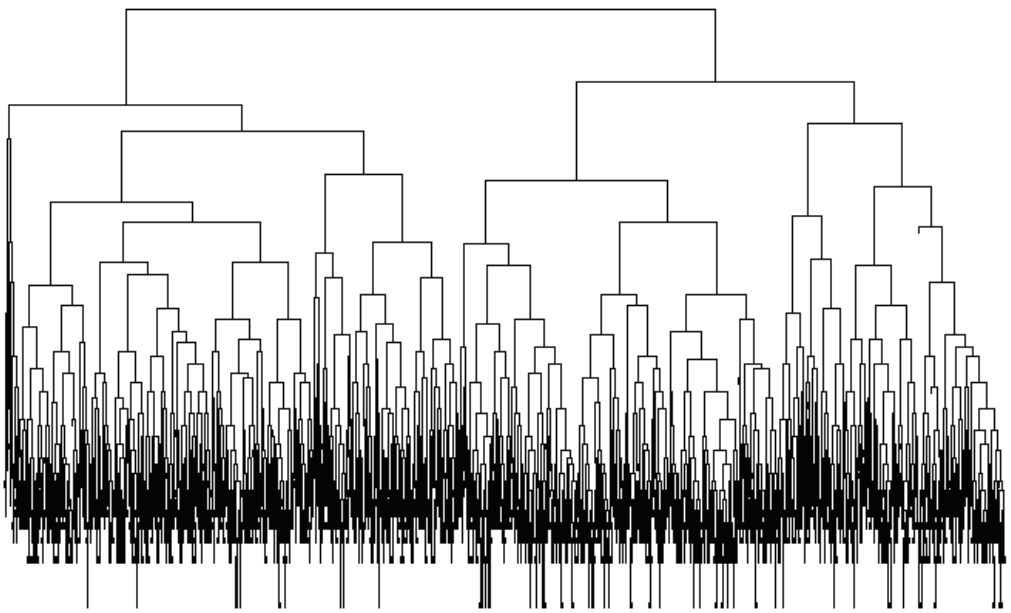

Figure 8: Dendrogram of the hierarchical clustering performed on the data.

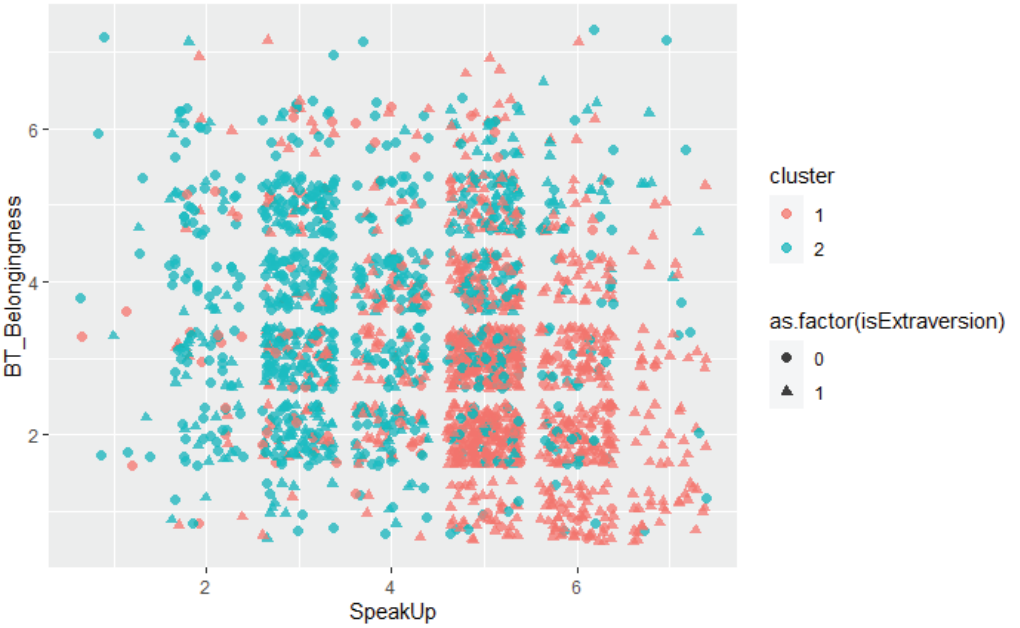

Figure 9: Scatterplot of the data points separated into 2 cluster.

Discussion

From the results, it can be observed that there are correlations between "SpeakUp" and "Extraversion" and "BT_BelongingConcern" and "Extraversion". This suggests that among the students who responded to the Beginning of Term survey, those students who identify as often holding back ideas or preferences to keep a group happy are also usually the students who would identify as tending to listen more than speak. Moreover, from the survey, I learnt that if students feel that they do not fit into the course even before the term has begun, these students are also likely to listen more than they speak in a team discussion. Nonetheless, even though the aforementioned correlations between variables exist, the correlation coefficient is not large enough to show a strong correlation between the variables. I suggest that in the future, the survey forms can include more questions on students' background so that future researchers can determine if there are other variables other than "SpeakUp" and "BT_BelongingConcern" that are affecting a student's "Extraversion" score.

From the second hypothesis, it is noticeable that among the students who responded to the survey, if a student has many teamwork experiences or has past positive teamwork experiences, then the student is expected to be more talkative and more likely to fit into a new course that contains teamwork projects. I believe that past experiences gave students the courage and confidence

to express themselves in teams and they no longer feel scared to communicate with others in the team. Owing to this, they expect themselves to fit into the new course more easily than other students who had no experiences. Nonetheless, there is no significant evidence showing that students with more experiences will speak up about their ideas or preferences even if it disrupts the group. I believe that for some students, the thought of interrupting other students' ideas is rude so they choose to hold back their ideas. This explanation is similar to Medaille and Usinger's statement of "silence in teams is the result of personal, social, academic, cultural and contextual constraints" (243).

It is important to note several limitations of this study. First, the result of the survey might not be a good representation of the students themselves. This is because sometimes the students might not accurately categorize themselves. For example, a high achieving student might have imposter syndrome and thus feel that he is doing badly and does not deserve to be on the team; or a student thinks that he or she is actually talkative, but in reality, he or she is quiet. Therefore, it is recommended to use an End-of-Term survey that includes both the students' evaluation of themselves and their peer evaluations on them.

Secondly, although all the respondents are students and all of their courses contain teamwork projects and discussion, the nature of the teamwork projects and discussions might not be the same across different disciplines. For example, a team discussion in humanities class might be interesting or relaxing while a team discussion in engineering class might be boring or stressful. As a result, students from different disciplines might have different attitudes towards the idea of teamwork and collaborative learning. Future research can be more precise by focusing on investigating whether the above results still hold in each discipline (Humanities, Social Sciences, Engineering, etc.)

Thirdly, the results and predictions obtained in this study are only applicable at the beginning of each course term. When the courses begin, there are even more factors throughout the semesters that may change a student's attitude such as the quality of the lecturers, the course's syllabus, and the quality of the peer discussions. It is suggested that future research can also focus on investigating how different factors that happen throughout the semester might change a student's personality (such as from being quiet to talkative).

Lastly, it is to be noted that this survey focuses on students' data collected during Fall 2020 and the result cannot be used to predict future students' personality who enrolled in those classes unless a new survey form is filled out by the students and the same analytical method is performed. This result is useful in helping instructors to predict a student's personality in the beginning of the semester, but it does not guarantee that the prediction is always accurate as the student's personality changes throughout the semester. Therefore, an instructor

should always observe any changes in students throughout the semester and make suitable changes to the group arrangement if necessary.

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UMURJ
University of Michigan Undergraduate Research Journal

Estimates of the Bounds of $\pi(x)$ and $\pi((x+1)^2) - \pi(x^2)$

CONNOR PAUL WILSON

We show the following bounds on the prime counting function $\pi(x)$ using principles from analytic number theory, giving an estimate

$$\log 2 \leq \liminf_{x\to\infty} \frac{\pi(x)}{x/\log x} \leq \limsup_{x\to\infty} \frac{\pi(x)}{x/\log x} \leq 2\log 2$$

We also conjecture about the bounding of $\pi((x+1)^2) - \pi(x^2)$, as is relevant to Legendre's conjecture about the number of primes in the aforementioned interval such that:

$$\left\lfloor \frac{1}{2}\left(\frac{(x+1)^2}{\log(x+1)} - \frac{x^2}{\log x} \right) - \frac{(\log x)^2}{\log(\log x)} \right\rfloor \leq \pi\left((x+1)^2\right) - \pi\left(x^2\right) \leq \left\lfloor \frac{1}{2}\left(\frac{(x+1)^2}{\log(x+1)} - \frac{x^2}{\log x} \right) - \log^2 x \log\log x \right\rfloor$$

1. Introduction

Recall the definition:

$$\pi(x) := \sum_{\substack{p \leq x \\ p \text{ prime}}} 1,$$

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and let us define the following:

$$\Theta(x) := \sum_{\substack{p \le x \\ p \text{ prime}}} \log p,$$

$$\Psi(x) := \sum_{1 \le n \le x} \Lambda(n) = \sum_{\substack{p^m \le x \\ m \ge 1 \\ p \text{ prime}}} \log p,$$

$$\Lambda(n) = \begin{cases} \log p & \text{if } n = p^k \text{ for some prime } p \text{ and integer } k \ge 1 \\ 0 & \text{otherwise.} \end{cases}$$

The following are simple statements from real analysis that are required for rigorousness' sake: let $\{x_n\}$ be a sequence of real numbers and L be a real number with the following two properties: $\forall \epsilon > 0$, $\exists N$ such that $x_n < L + \epsilon$, $\forall n \ge N$. $\forall \epsilon > 0 \wedge N \ge 1$, $\exists n \ge N$ with $x_n > L - \epsilon$. We thus define L as:

$$\limsup_{n \to \infty} x_n = L$$

Thus on the contrary we must have:

$$\liminf_{n \to \infty} x_n = -\limsup_{n \to \infty} -x_n$$

2. Necessary Preliminary Results

Theorem 2.1. *For all $\alpha \in (0, 1)$, and all $x \ge x_0$:*

$$\frac{\Theta(x)}{\log(x)} \le \frac{\Psi(x)}{\log(x)} \le \pi(x) \le \frac{\Theta(x)}{\alpha \log(x)} + x^{\alpha}$$

Proof. Clearly $\Theta(x) \le \Psi(x)$, such that

$$\limsup_{x \to \infty} \frac{\Psi(x)}{x} \ge \limsup_{x \to \infty} \frac{\Theta(x)}{x}$$

Also, if p is a prime and $p^m \le x < p^{m+1}$, then $\log p$ occurs in the sum for $\Psi(x)$ exactly m times. [1]

$$\begin{aligned}\Psi(x) &= \sum_{\substack{p^m \le x \\ p \text{ prime} \\ m \ge 1}} \log p \\ &= \sum_{\substack{p \le x \\ p \text{ prime}}} \left[\frac{\log x}{\log p} \right] \log p \\ &\le \sum_{\substack{p \le x \\ p \text{ prime}}} \log x \\ &= \pi(x) \log x\end{aligned}$$

$$\limsup_{x \to \infty} \frac{\Psi(x)}{x} \le \limsup_{x \to \infty} \frac{\pi(x)}{x / \log x}$$

Now fix $\alpha \in (0, 1)$. Given $x > 1$,

$$\Theta(x) = \sum_{\substack{p \le x \\ p \text{ prime}}} \log p \ge \sum_{\substack{x^\alpha < p \le x \\ p \text{ prime}}} \log p .$$

It is clear that all p from the second sum satisfy: $\log p > \alpha \log x$.

$\therefore$

$$\begin{aligned}\Theta(x) &> \alpha \log x \sum_{\substack{x^\alpha < p \le x \\ p \text{ prime}}} 1 \\ &= \alpha \log x \left(\pi(x) - \pi\left(x^\alpha\right) \right) \\ &> \alpha \log x \left(\pi(x) - x^\alpha \right)\end{aligned}$$

э

$$\frac{\Theta(x)}{x} > \frac{\alpha \pi(x)}{x / \log x} - \frac{\alpha \log x}{x^{1-\alpha}}$$

$\forall \alpha \in (0, 1)$ we have:

$$\lim_{x \to \infty} \frac{\alpha \log x}{x^{1-\alpha}} = 0.$$

Combining these we get:

$$\limsup_{x\to\infty}\frac{\Theta(x)}{x}\geq\limsup_{x\to\infty}\frac{\alpha\pi(x)}{x/\log x}$$

Once again, since our statement is true $\forall\alpha\in(0,1)$,

$$\limsup_{x\to\infty}\frac{\Theta(x)}{x}\geq\limsup_{x\to\infty}\frac{\pi(x)}{x/\log x}$$

Similarly:

$$\liminf_{x\to\infty}\frac{\Psi(x)}{x}\geq\liminf_{x\to\infty}\frac{\Theta(x)}{x}$$

$$\liminf_{x\to\infty}\frac{\pi(x)}{x/\log x}\geq\liminf_{x\to\infty}\frac{\Psi(x)}{x}$$

Once again, we apply the same method:

$$\liminf_{x\to\infty}\frac{\Theta(x)}{x}\geq\liminf_{x\to\infty}\frac{\pi(x)}{x/\log x},$$

and have thus proven **Theorem 2.1**. □

3. Main Result

Theorem 3.1. *We have:*

$$\log 2\leq\liminf_{x\to\infty}\frac{\pi(x)}{x/\log x}\leq\limsup_{x\to\infty}\frac{\pi(x)}{x/\log x}\leq 2\log 2.$$

Proof. First the lower bound. Take:

$$S(x):=\sum_{1\leq n\leq x}\log n-2\sum_{1\leq n\leq x/2}\log n.$$

Then,

$$\begin{aligned}\sum_{\substack{1\le d\le n\\ d\mid n}}\Lambda(d) &= \sum_{\substack{p^r\mid d\\ d\mid n\\ p\,\text{prime}}}\log p\\ &= \sum_{i=1}^{l}\sum_{r=1}^{e_i}\log p_i \quad \text{where } n = p_1^{e_1}\cdots p_l^{e_l}\\ &= \sum_{i=1}^{l} e_i \log p_i\\ &= \log n\end{aligned}$$

$\therefore$

$$S(x) = \sum_{1\le n\le x}\sum_{d\mid n}\Lambda(d) - 2\sum_{1\le n\le x/2}\sum_{d\mid n}\Lambda(d)$$

Clearly $\{d, 2d, \ldots, qd\}$ is the set of n satisfying $1 \le n \le x$ and $d \mid n$ (we can see this easily by writing $x = r + qd$ with $0 \le r < d$).

$\therefore$

$$\begin{aligned}S(x) &= \sum_{1\le d\le x}\Lambda(d)\left[\frac{x}{d}\right] - 2\sum_{1\le d\le x/2}\Lambda(d)\left[\frac{x}{2d}\right]\\ &= \sum_{1\le d\le x/2}\Lambda(d)\left(\left[\frac{x}{d}\right] - 2\left[\frac{x}{2d}\right]\right) + \sum_{(x/2)<d\le x}\Lambda(d)\left[\frac{x}{d}\right]\\ &\le \sum_{1\le d\le x/2}\Lambda(d) + \sum_{(x/2)<d\le x}\Lambda(d)\\ &= \Psi(x).\end{aligned}$$

So,

$$\frac{\Psi(x)}{x} \ge \frac{S(x)}{x} = \frac{1}{x}\sum_{1\le n\le x}\log n - \frac{2}{x}\sum_{1\le n\le x/2}\log n.$$

$\log t$ is increasing,

$\therefore$

$$\int_1^{x+1}\log t\,dt \ge \sum_{1\le n\le x}\log n,$$

$$\int_1^{[x]}\log t\,dt \le \sum_{1\le n\le x}\log n.$$

Actually, assuming $x \in \mathbb{Z}^+$,

$$\begin{aligned}\frac{S(x)}{x} &\geq \frac{1}{x}\int_1^x \log t\,dt - \frac{2}{x}\int_1^{(x/2)+1} \log t\,dt \\ &= \frac{1}{x}\left(x\log x - x + 1\right) - \frac{2}{x}\left(\frac{x+2}{2}\log\left(\frac{x+2}{2}\right) - \frac{x+2}{2} + 1\right) \\ &= \log x + \frac{1}{x} - \frac{x+2}{x}\log\left(x+2\right) + \frac{x+2}{x}\log 2 \\ &> \log\left(\frac{x}{x+2}\right) - \frac{2}{x}\log\left(x+2\right) + \log 2\end{aligned}$$

Using **Theorem 2.1**, we get:

$$\begin{aligned}\liminf_{x\to\infty} \frac{\pi(x)}{x/\log x} &= \liminf_{x\to\infty} \frac{\Psi(x)}{x} \\ &\geq \liminf_{x\to\infty} \frac{S(x)}{x} \\ &> \lim_{x\to\infty} \log\left(x/(x+2)\right) - \frac{2}{x}\log\left(x+2\right) + \log 2 \\ &= \log 2\end{aligned}$$

To complete the proof, we will need some auxiliary results taken from Murty's *Analytic Number Theory* [1] in the form of three lemmas:

Lemma 3.2.

$$\operatorname{ord}_p(m!) = \sum_{r\geq 1}\left\lfloor \frac{m}{p^r} \right\rfloor, \forall m \in \mathbb{Z}^+,\ \textit{prime } p$$

Proof. Fix an exponent r. The positive integers no larger than m that are multiples of p^r are

$$p^r, 2p^r, \ldots, \left\lfloor \frac{m}{p^r} \right\rfloor p^r$$

and those that are multiples of p^{r+1} are

$$p^{r+1}, 2p^{r+1}, \ldots, \left\lfloor \frac{m}{p^{r+1}} \right\rfloor p^{r+1}$$

Thus there are precisely $\left\lfloor m/p^r \right\rfloor - \left\lfloor m/p^{r+1} \right\rfloor$ positive integers $n \le m$ with $\mathrm{ord}_p(n) = r$.

$\therefore$

$$\begin{aligned}
\mathrm{ord}_p(m!) &= \sum_{n=1}^{m} \mathrm{ord}_p(n) \\
&= \sum_{r\ge 1} \sum_{\substack{1\le n\le m \\ \mathrm{ord}_p(n)=r}} r \\
&= \sum_{r\ge 1} r\left(\left\lfloor m/p^r \right\rfloor - \left\lfloor m/p^{r+1} \right\rfloor\right) \\
&= \sum_{r\ge 1} r\left\lfloor m/p^r \right\rfloor - \sum_{r\ge 1} r\left\lfloor m/p^{r+1} \right\rfloor \\
&= \sum_{r\ge 1} r\left\lfloor m/p^r \right\rfloor - \sum_{r\ge 1} (r-1)\left\lfloor m/p^r \right\rfloor \\
&= \sum_{r\ge 1} \left\lfloor \frac{m}{p^r} \right\rfloor
\end{aligned}$$

□

Lemma 3.3. $\forall n \in \mathbb{Z}^+$,

$$\frac{2^{2n}}{2\sqrt{n}} < \binom{2n}{n} < \frac{2^{2n}}{\sqrt{2n+1}}$$

Proof.

$$\begin{aligned}
P_n &:= \prod_{i\le n} \frac{(2i-1)}{(2i)} \\
&= \frac{(2n)!}{2^{2n}(n!)^2} \\
&= \binom{2n}{2} \frac{1}{2^{2n}}
\end{aligned}$$

Since:

$$\frac{(2i-1)(2i+1)}{(2i)^2} < 1$$

for all $i \ge 1$.

$\therefore$

$$1 > (2n+1)P_n^2,$$

giving the upper bound. For the lower bound:

$$1-\frac{1}{(2i-1)^2}<1$$

$\forall i \geq 1$, such that

$$1>\prod_{i=2}^{n}\left(1-\frac{1}{(2i-1)^2}\right)$$

$$=\prod_{i=2}^{n}\frac{(2i-1)^2-1}{(2i-1)^2}$$

$$=\prod_{i=2}^{n}\frac{(2i-2)(2i)}{(2i-1)^2}$$

$$=\frac{1}{4nP_n^2}$$

yielding our lemma. □

Lemma 3.4. $\forall n \in \mathbb{Z}^+$,

$$\Theta(n)<2n\log 2$$

Proof. By **Lemma 3.3**,

$$\log\left(\binom{2n}{n}\frac{1}{2}\right)=\log\left(\binom{2n}{n}\right)-\log 2$$

$$<2n\log 2-\frac{1}{2}\log(2n+1)-\log 2$$

$$=(2n-1)\log 2-\frac{1}{2}\log(2n+1)$$

since

$$\binom{2n}{n}\frac{1}{2}=\frac{(2n)!}{(n!)^2}\frac{n}{2n}=\frac{(2n-1)!}{n!(n-1)!}=\binom{2n-1}{n-1}.$$

by **Lemma 3.2**:

$$\log\left(\binom{2n}{n}\frac{1}{2}\right)=\log\binom{2n-1}{n-1}$$

$$=\sum_{p\,\text{prime}}\operatorname{ord}_p\big((2n-1)!\big)\log p-\sum_{p\,\text{prime}}\operatorname{ord}_p\big((n-1)!\big)\log p-\sum_{p\,\text{prime}}\operatorname{ord}_p\big(n!\big)\log p$$

$$=\sum_{p\,\text{prime}}\log p\sum_{r\geq 1}\Big[(2n-1)/p^r\Big]-\Big[n/p^r\Big]$$

$$\geq\sum_{\substack{p\,\text{prime}\\ n<p\leq 2n-1}}\log p$$

$$=\Theta\big(2n-1\big)-\Theta\big(n\big)$$

Where

$$\Theta\big(2n-1\big)-\Theta\big(n\big)<\big(2n-1\big)\log 2-\frac{1}{2}\log\big(2n+1\big)$$

We now proceed by induction. Proceeding from the trivialities, suppose $m>2$ and the lemma is true for $n<m, n, m\in\mathbb{N}$. If m is odd, then $m = 2n - 1$ for some integer $n \geq 2$ since $m > 2$. Thus by induction,

$$\Theta(m) = \Theta(2n-1)<\Theta(n) + (2n-1)\log 2-\frac{1}{2}\log\big(2n+1\big)$$

$$<2n\log 2 + (2n-1)\log 2-\frac{1}{2}\log(2n)$$

$$=(4n-1)\log 2-\frac{1}{2}\log(2n)$$

$$\leq(4n-2)\log 2 \qquad (\textit{since}\, n\geq 2)$$

$$=2m\log 2$$

If m is even, then $m = 2n$ for some integer n with $m > n \geq 2$ and m is composite. Clearly $\Theta(m) = \Theta(m - 1)$ and we know:

$$\Theta(m) = \Theta(m-1)<2(m-1)\log 2<2m\log 2$$

Lemma 3.4 gives

$$2\log 2 \geq \limsup_{x\to\infty} \frac{\Theta(x)}{x}. \qquad \square$$

The desired lower bound follows from **Theorem 2.1** $\square$

4. On Primes in the Gaps between Squares

The following is relatively aleatory compared to the previous workings, but it is worth mentioning considering the importance of the statement.

By Hassani [2], we have

$$\left\lfloor \frac{1}{2}\left(\frac{(x+1)^2}{\log(x+1)} - \frac{x^2}{\log x} \right) - \frac{\log^2 x}{\log\log x} \right\rfloor \leq \pi\left((x+1)^2\right) - \pi\left(x^2\right)$$

$$\frac{1}{2}\left(\frac{x^2}{\log n} - \frac{3^2}{\log 3} \right) - \sum_{j=3}^{x-1} \frac{\log^2 x}{\log\log k} < \pi\left(x^2\right) - \pi\left(3^2\right)$$

And thus:

$$\sum_{j=3}^{x-1} \left\lfloor \frac{1}{2}\left(\frac{(j+1)^2}{\log(j+1)} - \frac{j^2}{\log j} \right) - \frac{\log^2 j}{\log\log j} \right\rfloor < \sum_{j=3}^{n-1} \pi\left((j+1)^2\right) - \pi\left(j^2\right)$$

$\therefore$

$$\left\lfloor \frac{1}{2}\left(\frac{(x+1)^2}{\log(x+1)} - \frac{x^2}{\log x} \right) - \frac{(\log x)^2}{\log(\log x)} \right\rfloor \leq \pi\left((x+1)^2\right) - \pi\left(x^2\right).$$

And by the prime number theorem, which gives us the asymptotic estimate for some

$$F(x) := \pi\left((x+1)^2\right) - \pi\left(x^2\right) \sim \frac{1}{2}\left(\frac{(x+1)^2}{\log(x+1)} - \frac{x^2}{\log x} \right)$$

We propose:

$$\pi\left((x+1)^2\right) - \pi\left(x^2\right) \leq \left\lfloor \frac{1}{2}\left(\frac{(x+1)^2}{\log(x+1)} - \frac{x^2}{\log x} \right) + \log^2 x \log\log x \right\rfloor$$

by the same method.

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1. R. Ram Murty, *Problems in Analytic Number Theory, Second Edition — Graduate Texts in Mathematics*, Springer, 2001.
2. M. Hassani, *Counting primes in the interval* $(n^2, (n+1)^2)$, AMS, 1997.

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